EXAM 1 — TQS 308 — Fall 2010

NAME:

True/False: If the statement is *always* true, give a brief explanation of why it is. If the statement is false, give a counterexample.

1. [4] For every vector $\overrightarrow{u}$, $\overrightarrow{v}$ and $\overrightarrow{w}$ in $\mathbb{R}^n$

$$(\overrightarrow{u} + \overrightarrow{v}) + \overrightarrow{w} = \overrightarrow{u} + (\overrightarrow{v} + \overrightarrow{w}).$$

2. [4] For every vector $\overrightarrow{u}$, $\overrightarrow{v}$, and scalar c

$$c \cdot (\overrightarrow{u} + \overrightarrow{v}) = c \cdot \overrightarrow{u} + c \cdot \overrightarrow{v}.$$

3. [4] The vectors $||\overrightarrow{u} + \overrightarrow{v}|| = ||\overrightarrow{u}|| + ||\overrightarrow{v}||$ if and only if $\overrightarrow{u} = \overrightarrow{0}$.

4. [4] The following equation defines a plane in $\mathbb{R}^3$:

$$\begin{bmatrix}1\\1\\1\end{bmatrix} + t\begin{bmatrix}1\\2\\3\end{bmatrix} + r\begin{bmatrix}3\\6\\9\end{bmatrix}$$

5. [4] Assume you are given a matrix with at least two rows. Perform R_2+R_1 and R_1+R_2. Now rows 1 and 2 are identical. Now perform $R_2 - R_1$ to obtain a row of zeros in the second row. Thus all matrixes with at least two rows are row equivalent to a matrix with a zero row.

6. [4] The span of

$$\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \text{and} \begin{bmatrix}0\\0\\1\end{bmatrix}$$

is all of $\mathbb{R}^3$.

Free Responce: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

7. [8] Use Gaussian or Gauss-Jordan elimination to find solution(s) if they exist, to the following system of linear equations: $\left\{\begin{array}{c} -2x_1 + 2x_2 = -10 \\ 6x_1 + 2x_2 + 2x_3 = 14 \end{array}\right\}$

8. [2] Find a reduced row echelon (or row echelon) form for A where $A = \begin{bmatrix} -2 & 2 & 0 & -10 \\ 6 & 2 & 2 & 14 \end{bmatrix}$

   (a) Verify the Rank Theorem by doing the following:

      i. [1] Find rank(A).

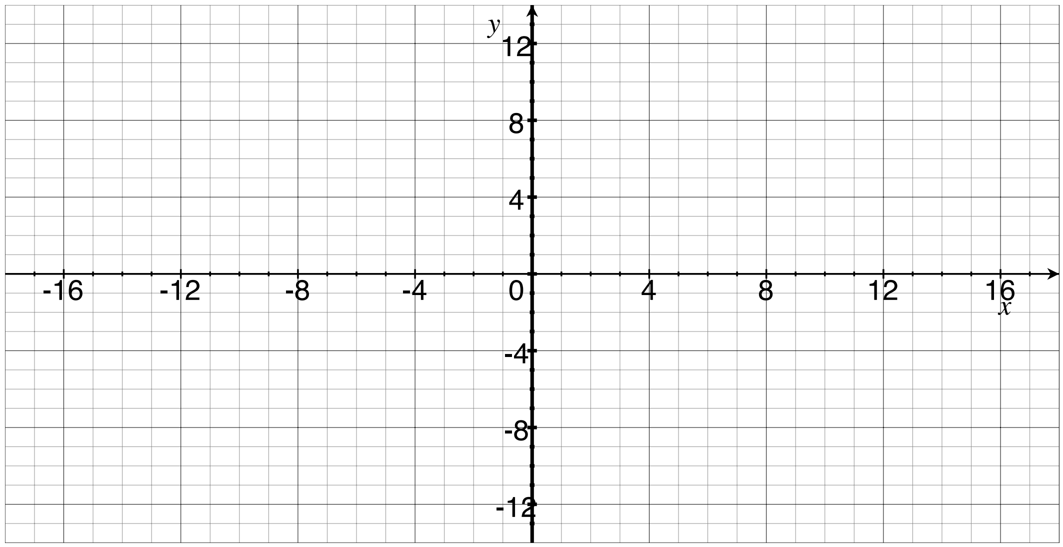
      ii. [1] How many variables are there?

      iii. [1] Does the number of free variables=the number of variables -rank(A)?

9. Let the following vectors be in $\mathbb{R}^2$.

$$\overrightarrow{u} = [-2, 6] \qquad \overrightarrow{v} = [2, 2] \qquad \overrightarrow{w} = [0, 2]$$

(a) [4] Draw the vector $\overrightarrow{u} + 2\overrightarrow{v} + \overrightarrow{w}$ on the axes below.



(b) [4] Is the angle between $\overrightarrow{u}$ and $-\overrightarrow{v}$, obtuse, acute or right?

(c) [5] Is $\overrightarrow{z} = [-10, 14]$ a linear combination of $\overrightarrow{u}$, $\overrightarrow{v}$, and $\overrightarrow{w}$? If so write $\overrightarrow{z}$ as a linear combination of $\overrightarrow{u}$, $\overrightarrow{v}$, and $\overrightarrow{w}$. (Consider using work from the previous page.)

(d) [1] Verify your answer to (d) graphically.

10. Let the $\overrightarrow{u} = [-2, 5, -5]$ be in $\mathbb{R}^3$.

(a) [2] Write the vector form for a line that is parallel to $\overrightarrow{u}$ and passes through $(3, 2, 1)$.

(b) [8] Let $\mathcal{L}$ be the line that is described in (a). Find the distance from the point $Q = (1, 0, 2)$ to the line $\mathcal{L}$.

11. [9] *Prove* vectors $\overrightarrow{v_1}$, $\overrightarrow{v_2}$, ... $\overrightarrow{v_k}$ are linearly dependent if and only if at least one of the vectors can be expressed as a linear combination of the others.