Quiz 8
Math 253

Name: Key

TRUE/FALSE: Write "True" in each of the following cases if the statement is *always* true and give justification. Otherwise, write "False" and provide a counterexample.

3 1. [3] Given that $\sum_{n=0}^{\infty} c_n(-4)^n$ is convergent, $\sum_{n=0}^{\infty} c_n 4^n$ also converges.

(+1) false let $c_n = \left(\frac{1}{4}\right)^n \cdot \frac{1}{n}$

(+2)

then $\sum_{n=0}^{\infty} c_n(-4)^n = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n \frac{1}{n}(-4)^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{n}$ alt harmonic conv.

but $\sum_{n=0}^{\infty} c_n(4)^n = \sum_{n=0}^{\infty} \cancel{\left(\frac{1}{4}\right)^n} \frac{1}{n} \cancel{(4)^n} = \sum_{n=0}^{\infty} \frac{1}{n}$ harmonic div

2. [2] Given that $\sum_{n=0}^{\infty} c_n(-4)^n$ is convergent, $\sum_{n=0}^{\infty} c_n$ also converges.

Note if we were given $\sum_{n=0} c_n x^n$ is convergent for $x=-4$ this would have been true without much work.

As is it is... true.

Since $\sum_{n=0}^{\infty} c_n(-4)^n$ converges $\lim_{n\to\infty} c_n(-4)^n = 0$

$\Rightarrow \lim_{n\to\infty} |4^n c_n| = 0$

Thus for some large N, $n > N \Rightarrow |4^n c_n| < 1$

$\Rightarrow |c_n| < \frac{1}{4^n}$

$\Rightarrow$ the tail is smaller than a geometric series that is known to converge.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

[4] 3. [X] Find the interval of convergence for $\sum_{n=2}^{\infty}(-1)^n \frac{(2x+3)^n}{n \ln n}$

use the ratio test so abs. conv if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{(2x+3)^{n+1}}{(n+1)\ln(n+1)}}{\frac{(2x+3)^n}{n\ln n}} = \frac{(2x+3)^{n+1}\, n\ln n}{\cancel{(2x+3)^n}\,(n+1)\ln(n+1)} = (2x+3)\frac{n\ln n}{(n+1)\ln(n+1)}$$

notation/alg (+1)
looking for radius (+1)
looking for end (+1)
justified limit (+1)

$$\lim_{n\to\infty}\left|(2x+3)\frac{n\ln n}{(n+1)\ln(n+1)}\right| = |2x+3| \lim_{n\to\infty}\left|\frac{n\ln n}{(n+1)\ln(n+1)}\right| \overset{L'H}{=} |2x+3|\lim_{n\to\infty}\left|\frac{1+\ln n}{1+\ln(n+1)}\right|$$

$$\overset{LH}{=} |2x+3|\lim_{n\to\infty}\left|\frac{\frac{1}{n}}{\frac{1}{n+1}}\right| = |2x+3|\lim_{n\to\infty}\left|\frac{n+1}{n}\right| = |2x+3|$$

$\therefore$ abs conv when $|2x+3| < 1 \Rightarrow -1 < 2x+3 < 1$
$-4 < 2x < -2$
$-2 < x < -1$

to check the endpoints: $(-2,-1]$

$x = -2 \Rightarrow \sum_{n=2}^{\infty} \frac{1}{n\ln n}$

integral test $\int_2^{\infty} \frac{1}{n\ln n}\,dn = \int_{\ln 2}^{\ln(\infty)} \frac{1}{u}\,du$ div

$x = -1 \Rightarrow \sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$

$\lim_{n\to\infty} \frac{1}{n\ln n} = 0$

$\frac{d}{dx}\left(\frac{1}{n\ln n}\right) = \frac{-1}{(n\ln n)^2}(1+\ln n)$

$= \frac{-1-\ln n}{(n\ln n)^2}$

< 0 conv

[3] 4. [X] Find a power series representation for

$$\frac{1}{1-(x^3)} \quad : \quad f(x) = \frac{1}{1-x^3}.$$

notation (+1)
right (+1)

$1 + (x^3) + (x^3)^2 + (x^3)^3 + \ldots = \sum_{n=0}^{\infty} (x^3)^n$

$1 + x^3 + x^6 + x^9 + \ldots = \sum_{n=0}^{\infty} x^{3n}$

since

$\frac{1}{1-u} = \sum_{n=0}^{\infty} u^n$