Quiz 7
Math 253

Name: KEY

Directions: Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Clearly show your work and say what techniques and reasoning you used. If it is convergent write as much information as you can about the limit.

1. [3] $\sum_{n=2}^{\infty} \frac{1}{n^2 - n}$

$\frac{a}{n} + \frac{b}{n-1} = \frac{1}{n^2-n}$ $\qquad$ $\frac{1}{n-1} \cdot \frac{1}{n} = \frac{1}{n(n-1)}$ ✓

$an - a + bn = 1$

$a = -1 \Rightarrow b = 1$

$S_k = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots + \left(\frac{1}{k-1} - \frac{1}{k}\right) = 1 - \frac{1}{k}$

$\lim_{k\to\infty} S_k = \lim_{k\to\infty}\left(1 - \frac{1}{k}\right) = 1$

abs convergent b/c $\left|\frac{1}{n^2-n}\right| = \frac{1}{n^2-n}$

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2. [3] $\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$

$\frac{\frac{(n+1)!}{e^{(n+1)^2}}}{\frac{n!}{e^{n^2}}} = \frac{(n+1)n!\, e^{n^2}}{n!\, e^{n^2+2n+1}} = \frac{n+1}{e^{2n+1}}$

$\lim_{n\to\infty} \frac{n+1}{e^{2n+1}} = \lim_{x\to\infty} \frac{x+1}{e^{2x+1}} \overset{L'H}{=} \lim_{x\to\infty} \frac{1}{2e^{2x+1}} = 0 < 1$

$\therefore$ by the ratio test this abs. conv.

used a test (+1)
notation/alg (+1)
got it (+1)

notes
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3. [3] $\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{4n-1}$

used a test
notation/alg
get it.

$$\lim_{n\to\infty} \frac{3n}{4n-1} = 3/4 \neq 0$$

and $\lim_{n\to\infty} \frac{(-1)^n 3n}{4n-1}$ DNE $\Rightarrow$ by connection between seq & series, this diverges.

notes #24
§11.6

4. [3] $\sum_{n=1}^{\infty} \frac{(-1)^n}{(\arctan n)^n}$

$$\lim_{n\to\infty} \left|\sqrt[n]{\frac{1}{(\arctan n)^n}}\right| = \lim_{n\to\infty} \left|\frac{1}{\arctan n}\right| = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi} < 1$$

abs conv by root test.