Quiz 6
Math 253

give 20min

Name:

:11 8 min

Directions: Determine whether the series is convergent or divergent. Clearly show your work and say what techniques you used. If it is convergent write as much information as you can about the limit.

#9 HW pg 734 §11.4

1. [3] $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$

$0 \le \cos^2 n \le 1$

$\Rightarrow \frac{\cos^2 n}{n^2+1} \le \frac{\cos^2 n}{n^2} \le \frac{1}{n^2}$

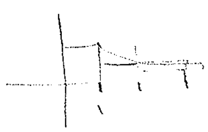
$n^2+1 \ge n^2 \Rightarrow \frac{1}{n^2+1} \le \frac{1}{n^2}$

But $\sum \frac{1}{n^2}$ converges by the p-test

Since all the terms are postive the comparison test implies it converges.

Test (+1)

Sense (+1)



~~Terraroms~~ We can say $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1} \le \sum_{n=1}^{\infty} \frac{1}{n^2} \le 1 + \int_1^{\infty} \frac{1}{x^2}\,dx$

Example 9 pg 719 §11.2

2. [4] $\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)} + \frac{1}{2^n}\right) = \sum_{n=1}^{\infty} \frac{3}{n(n+1)} + \sum_{n=1}^{\infty} \frac{1}{2^n} = 3 + 1 = 4$

telescoping geom

$\sum_{n=1}^{\infty} \frac{3}{n(n+1)} = \sum_{n=1}^{\infty}\left(\frac{3}{n} + \frac{-3}{n+1}\right)$ using alg.

$\frac{a}{n} + \frac{b}{n+1} = \frac{3}{n(n+1)} \Rightarrow an + a + bn = 3 \quad a = 3 \quad b = -3$

Alg (+1)

Test (+1)

~~Teste~~ (+1)

~~sense~~ (+1)

$S_k = \left(\frac{3}{1} - \cancel{\frac{3}{2}}\right) + \left(\cancel{\frac{3}{2}} - \cancel{\frac{3}{3}}\right) + (\quad) + \dots + \left(\cancel{\frac{3}{k}} - \frac{3}{k+1}\right)$

$= 3 - 3/(k+1)$

so $\lim_{k\to\infty} S_k = \lim_{k\to\infty} \left(3 - 3/(k+1)\right) = 3$

$\sum_{n=1}^{\infty} \frac{1}{2^n}$ is a geom series with $a = 1/2$ & $r = 1/2$ so converges to $\frac{a}{1-r} = \frac{1/2}{1/2} = 1$

#21 4w

§ 11.3

3. [3] $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

looks nice to integrate

Integral test:

Test (+1)

Sense (+1)

$$\int_2^\infty \frac{1}{x \ln x}\,dx \qquad \text{let } u = \ln x, \quad du = \frac{1}{x}\,dx$$

$$\int \frac{1}{x \ln x}\,dx = \int \frac{1}{u}\,du = \ln u + c = \ln(\ln x) + c$$

so $$\int_2^\infty \frac{1}{x \ln x}\,dx = \ln(\ln x)\Big|_2^\infty$$

$$= \underbrace{\lim_{x\to\infty} \ln(\ln(x))}_{\text{div.}} - \underbrace{\ln(\ln 2)}_{\text{finite}}$$

The series diverges