Quiz 5
Math 253

Name: Key

TRUE/FALSE: Write "TRUE" in each of the following cases if the statement is *always* true and then *mathematically prove* it. Otherwise, write "FALSE" and provide a counterexample.

1. [2] If $\{a_n\}$ is a sequence such that $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is convergent.

(+1) False :

harmonic series: $\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

(+1) note $\lim_{n\to\infty} \frac{1}{n} = 0$

but $\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ 4 terms $< 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots$

so it diverges.

2. [2] If f is a continuous positive decreasing function on $[1, \infty)$ and $a_n = f(n)$ for all integers $n \geq 1$ and $\int_1^{\infty} f(x)\,dx$ diverges, then $\sum_{n=1}^{\infty} a_n$ also diverges.

(+1) True. Since f is a cont positive dec. function

right hand approx $< \int_1^n f(x)dx <$ left hand approx

$\Rightarrow \sum_{i=1}^{n-1} a_i < \int_1^n f(x)dx < \sum_{i=2}^{n} a_i$

(+1) Since $\lim_{n\to\infty} \int_1^n f(x)dx$ diverges.

$\int_1^n f(x)dx < \sum_{i=2}^{n} a_i < \sum_{i=1}^{n} a_i$

$\Rightarrow \lim_{n\to\infty} \int_1^n f(x)dx < \lim_{n\to\infty} \sum_{i=1}^{n} a_i$

$\Rightarrow \sum a_i$ diverges

Show *all* your work (algebraically or geometrically) for each and simplify. No credit is given without supporting work.

3. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

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(a) [3] $-2+\frac{5}{2}-\frac{25}{8}+\frac{125}{32}-\dots$ $=\sum_{n=1} -2\left(\frac{-5}{4}\right)^{n-1}$

geom series (+1)

$a=-2$

$r=-5/4$ (+1)

but $|-5/4|>1$

$\Rightarrow$ diverges (+1)

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(b) [3] $\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)}+\frac{1}{2^n}\right)$ $=\sum_{n=1}^{\infty}\frac{3}{n(n+1)}+\sum_{n=1}^{\infty}\frac{1}{2^n}$ split (+1)

(+1) $\sum\frac{1}{2^n}$ is a geom series $=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots$ $a=\frac{1}{2}$, $r=\frac{1}{2}$ so conv to $\frac{\frac{1}{2}}{1-\frac{1}{2}}=\frac{\frac{1}{2}}{\frac{1}{2}}=1$

(+1) $\sum\frac{3}{n(n+1)}$ probably some canceling trick:

PFD: $\frac{a}{n}+\frac{b}{n+1} \Rightarrow an+a+bn=3$

$(a+b)n=0$ $a=3$

$a=-b$

$3n+3-3n$ ✓

$\frac{3}{n}+\frac{-3}{n+1}$ ok

$=\sum_{n=1}^{\infty}\left(\frac{3}{n}-\frac{3}{n+1}\right)=\left(\frac{3}{1}-\cancel{\frac{3}{2}}\right)+\left(\cancel{\frac{3}{2}}-\cancel{\frac{3}{3}}\right)+\left(\cancel{\frac{3}{3}}-\cancel{\frac{3}{4}}\right)+\left(\cancel{\frac{3}{4}}-\frac{3}{5}\right)+\dots$

note $S_k=3-\frac{3}{k+1}$ so $\lim_{k\to\infty}S_k=3$

converges to 3

$\therefore \sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)}+\frac{1}{2^n}\right)=3+1=4.$