Quiz 5
Math 253

Name:

TRUE/FALSE: Write "TRUE" in each of the following cases if the statement is *always* true and then *mathematically prove* it. Otherwise, write "FALSE" and provide a counterexample.

1. [2] If $\{a_n\}$ is a sequence such that $\lim\limits_{n\to\infty} a_n = 0$, then $\sum\limits_{n=1}^{\infty} a_n$ is convergent.

2. [2] If f is a continuous positive decreasing function on $[1, \infty)$ and $a_n = f(n)$ for all integers $n \geq 1$ and $\int_1^{\infty} f(x)\,dx$ diverges, then $\sum\limits_{n=1}^{\infty} a_n$ also diverges.

Show *all* your work (algebraically or geometrically) for each and simplify. No credit is given without supporting work.

3. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

(a) [3] $-2 + \frac{5}{2} - \frac{25}{8} + \frac{125}{32} - \ldots$

(b) [3] $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{n(n+1)} + \frac{1}{2^n} \right)$