

# Quiz 4

## Math 253

Name: KEY

Show *all* your work (algebraically or geometrically) for each and simplify. No credit is given without supporting work.

1. [2] Explain carefully what ~~is~~ the difference between a sequence and a series is.

A sequence is an ordered list of numbers.

A series is the limit of a particular sequence - mainly the sequence of partial sums.

For instance if  $\{a_n\}$  is a sequence

The series denoted  $\sum a_n$  is

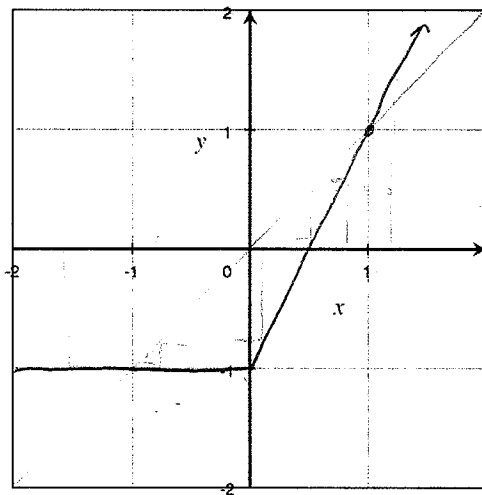
$$\lim_{n \rightarrow \infty} (a_1 + a_2 + \dots + a_n) \\ = \lim_{n \rightarrow \infty} \left( \sum_{i=1}^n a_i \right)$$

2. [2] If  $|x| < 1$ , what is  $\sum_{n=0}^{\infty} x^n$ ?

$$\frac{1}{1-x}$$

3. Let  $\{a_n\}$  be the sequence defined by  $f(a_n) = a_{n+1}$ , where  $f(x) = \begin{cases} -1; & \text{if } x \leq 0 \\ 2x - 1 & \text{otherwise} \end{cases}$

(a) [1] Draw  $f(x)$  on the axis provided below.



(b) [1] If  $a_1 = -2$ , write out a few of the terms from the sequence  $\{a_n\}$ .

$$\{-2, -1, -1, -1, \dots\}$$

(c) [1] What values can we set  $a_1$  to and have  $\{a_n\}$  diverge?

$$a_1 > 1$$

(d) [3] Write down the values of  $a_1$  that lead the sequence  $\{a_n\}$  to converge and what the the corresponding limits are.

$$a_1 < 1 \Rightarrow \lim_{n \rightarrow \infty} a_n = -1$$

$$a_1 = 1 \Rightarrow \lim_{n \rightarrow \infty} a_n = 1$$