Quiz 3
Math 253

Name: KEY

Show *all* your work (algebraically or geometrically) for each and simplify. No credit is given without supporting work.

1. [4] Find an *equation of the line* that is tangent at $(1,1)$ to the parametric curve described by $x(t) = e^t$ and $y(t) = (t-1)^2$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2(t-1)}{e^t} \quad \text{(+2) formula der's.}$$

if $1 = x = e^t \Rightarrow t = 0$

$$m = \left.\frac{dy}{dx}\right|_{t=0} = \underbrace{\frac{2(-1)}{1} = -2}_{(+1)}$$

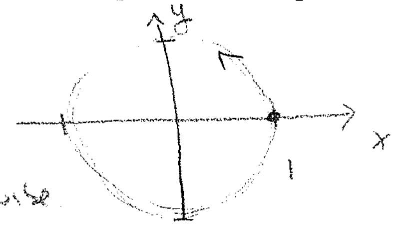
$$y - 1 = -2(x-1) = -2x + 2 \quad (+1)$$

$$y = -2x + 3$$

2. Let $x(\theta) = \cos\theta$ and $y(\theta) = \sin\theta$.

(a) [1] Describe or graph the curve that the above parametric equations describe for $0 \le \theta \le 2\pi$.

The unit circle (+1)
beginning at (1,0)
& moving counter clockwise



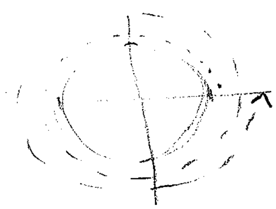
(b) [2] Use the techniques *from* §10.2 to compute the length of the curve described by the above parametric equations for $0 \le \theta \le 2\pi$.

$$\int_0^{2\pi} \sqrt{(-\sin\theta)^2 + (\cos\theta)^2}\, d\theta = \int_0^{2\pi} \sqrt{\sin^2\theta + \cos^2\theta}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{1}\, d\theta = x\Big|_0^{2\pi} = 2\pi$$

(c) [1] Describe or graph the curve that the above parametric equations describe for $0 \le \theta \le 4\pi$.

the unit circle
traversed twice (+1)
counterclockwise



(d) [2] Use *any* techniques you like to compute the length of the curve described by the above parametric equations for $0 \le \theta \le 4\pi$.

$2 \cdot 2\pi = 4\pi$