

# Quiz 2

## Math 253

Name:

Show *all* your work (algebraically or geometrically) for each and simplify. No credit is given without supporting work.

1. [2] Consider the function  $f(x) = x^2$  for any real  $x$ . Explain why this function has no inverse.

function is not one-to-one

$$f^{-1}(y) = x$$

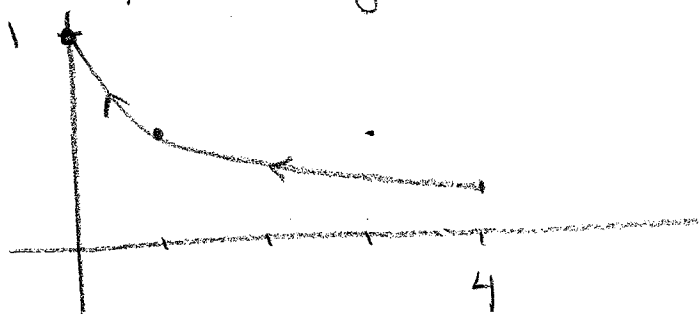
so if not 1-1  $y$  is sent to two places

2. [3] Consider the parametric equations  $x(t) = t^2$  and  $y(t) = e^t$  for  $-2 \leq t \leq 0$ .

Plot some points for

- Graph the corresponding parametric curve, and ~~try~~ guess at the shape

$t$	$x$	$y$
-2	4	$\frac{1}{e^2}$
-1	1	$\frac{1}{e}$
0	0	1



- Eliminate the parameter to find a Cartesian equation of the curve.

$$x = t^2 \Rightarrow t = -\sqrt{x}$$

$$y = e^{-\sqrt{x}}$$

3. [5] The curve  $y = \sqrt{4-x^2}$ ,  $-1 \leq x \leq 1$ , is an arc of the circle  $x^2 + y^2 = 4$ . Find the area of the surface obtained by rotating this arc about the  $x$ -axis.

$$\int_{-1}^1 2\pi \sqrt{4-x^2} \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2} dx$$

$$f(x) = y = \sqrt{4-x^2}$$

$$f'(x) = \frac{1}{\sqrt{4-x^2}} \cdot -2x$$

$$= \frac{-x}{\sqrt{4-x^2}}$$



$$= 2\pi \int_{-1}^1 \sqrt{4-x^2} \sqrt{\frac{4-x^2+x^2}{4-x^2}} dx$$

simplified  
but not all

$$= 2\pi \cdot 2 \int_{-1}^1 1 dx$$

$$= 4\pi \times \Big|_{-1}^1$$

$$= 8\pi \quad \text{①}$$