Quiz 1
Math 253

Name:

Show *all* your work (algebraically or geometrically) for each and simplify. No credit is given without supporting work.

1. [2] Explain when and why $\sqrt{1+\tan^2\theta} = \sec\theta$.

(+1) Pyth => $1+\tan^2\theta = \sec^2\theta$

So we're really considering when $\sqrt{\sec^2\theta} = \sec\theta$

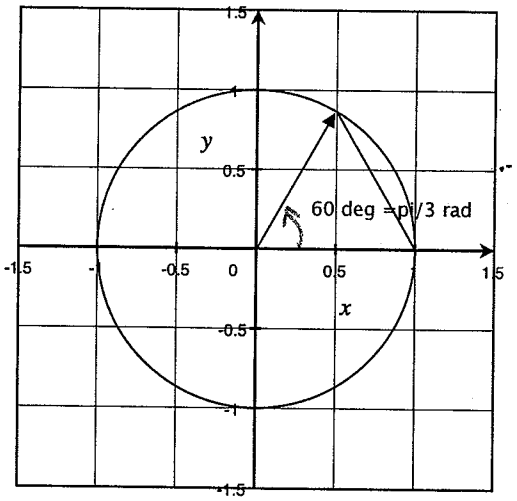
(+1) this happens when $\sec\theta > 0$

=> $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

2. [3] In case you don't remember, consider the diagram below and then find the following:

(+1) $\cos\frac{\pi}{3} = \frac{1}{2}$

(+1) $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$



Work is here (+1)

3. [5] Use the facts from the front to find the length of the curve $\ln(\cos x)$ from $0 \le x \le \frac{\pi}{3}$.

(+1) $\int_0^{\pi/3} \sqrt{1+(-\tan x)^2}\,dx$ (+2)

$f(x) = \ln(\cos x)$

$f'(x) = \frac{1}{\cos x} \cdot -\sin x$

$= -\tan x$

$= \int_0^{\pi/3} \sqrt{1+\tan^2 x}\,dx$

$= \int_0^{\pi/3} \sqrt{\sec^2 x}\,dx$ since $0 \le x < \frac{\pi}{2}$

$= \int_0^{\pi/3} \sec x\,dx$ notation (+1)

(+1) $= \ln|\sec x + \tan x|\Big|_0^{\pi/3}$

$= \ln|\sec\frac{\pi}{3} + \tan\frac{\pi}{3}| - \ln|\sec 0 + \tan 0|$

$= \ln|2+\sqrt{3}| - \ln|1+0|$

$= \ln(2+\sqrt{3})$