FINAL

Math 253

Practice

Note: This is a practice final and is intended only for study purposes. The actual exam will contain different questions and perhaps a different layout.

TRUE/FALSE: Write "True" in each of the following cases if the statement is *always* true and give justification. Otherwise, write "False" and provide a counterexample.

1. If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ is convergent.
2. If the series $\sum_{n=1}^{\infty}(a_n + b_n)$ is convergent, then both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent.
3. If both series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent, then $\sum_{n=1}^{\infty}(a_n + b_n)$ is convergent.
4. If both series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are convergent, then $\sum_{n=1}^{\infty}(a_n b_n)$ is convergent.
5. If both series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are divergent, then $\sum_{n=1}^{\infty}(a_n + b_n)$ is divergent.
6. If a series is absolutely convergent, then it converges.
7. If a series converges then it absolutely converges.
8. The series $\sum_{n=1}^{\infty} n^{-\sin 1}$ is convergent.
9. If $\sum c_n x^n$ diverges when $x = 6$, then it diverges when $x = 10$.
10. If $\sum c_n 6^n$ is convergent, then $\sum c_n(-6)^n$ is convergent.
11. If $0 \le a_n \le b_n$, and $\sum b_n$ diverges, then $\sum a_n$ diverges.
12. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{e}$.
13. If $\sum a_n$ is divergent, then $\sum |a_n|$ is divergent.
14. If $a_n > 0$ and $\sum a_n$ converges, then $\sum(-1)a_n$ converges.

15. If $a_n > 0$ and $\lim_{n\to\infty}(a_{n+1}/a_n) < 1$, then $\lim_{n\to\infty} a_n = 0$

16. If $\{a_n\}$ is divergent, then $\sum a_n$ is divergent.

17. If $\{a_n\}$ is convergent, then $\sum a_n$ is convergent.

18. If $\{a_n\}$ is convergent, then the series $\displaystyle\sum_{n=1}^{\infty}(a_{n+1} - a_n)$ is convergent.

19. If $\{a_n\}$ is divergent, then the series $\displaystyle\sum_{n=1}^{\infty}(a_{n+1} - a_n)$ is divergent.

20. If $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent, then $\displaystyle\sum_{n=m}^{\infty} a_n$ is convergent for every integer $m > 1$.

21. If m is an integer which is greater than one, and $\displaystyle\sum_{n=m}^{\infty} a_n$ is convergent, then $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent.

22. if $\sum a_n$ and $\sum b_n$ are both convergent series with positive terms, is it true that $\sum a_n b_n$ is also convergent?

23. If $\sum a_n$ is a convergent series with positive terms, is it ture that $\sum \sin(a_n)$ is also convergent?

FREE RESPONCE: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

24. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence.

    (a) Explain what $\displaystyle\sum_{n=1}^{\infty} a_n$ is to an eight year old.

    (b) What makes this explanation false?

    (c) What is the mathematical definition for $\displaystyle\sum_{n=1}^{\infty} a_n$.

25. Suppose $\sum a_n = 3$ and s_n is the nth partial sum of the series.

    (a) What is $\lim_{n\to\infty} a_n$?

    (b) What is $\lim_{n\to\infty} s_n$?

26. State the Limit Comparison Test.
    note: Other tests that are fair game: Test of Divergence, Integral Test, Comparison Test, Alternating Series Test, Ratio Test, and Root Test.

27. Determine whether the following converge or diverge. If one does convergent, find its sum.

    (a) $\{a_n\}$ where $a_n = \frac{9^{n+1}}{10^n}$.
    (b) $\{a_n\}$ where $a_n = \frac{n^3}{1+n^2}$.
    (c) $\{a_n\}$ where $a_0 = -\frac{1}{2}$ and $a_n = a_{n-1}^2$.
    (d) $\{a_n\}$ where $a_n = \cos(n\frac{\pi}{2})$.
    (e) $\{a_n\}$ where $a_n = \frac{(-10)^n}{n!}$.
    (f) $\{a_n\}$ where $a_n = \sqrt{n-1}\ln(\frac{2n^2}{n+2n^2})$.
    (g) $\displaystyle\sum_{n=1}^{\infty} 4^{-n}3^{2n}$
    (h) $\displaystyle\sum_{n=1}^{\infty}[\cos(\frac{1}{n}) - \cos(\frac{1}{n+1})]$
    (i) $\displaystyle\sum_{n=0}^{\infty} 3^n 2^{-2n}$
    (j) $\displaystyle\sum_{n=1}^{\infty} \ln(\frac{n^2}{(n+2)^2}$

28. Choose some random problems out of §11.7 between 1 and 38.

29. Choose some random problems out of §11.8 between 3 and 28.

30. Find the power series representation and the radius of convergence.

    $f(x) = \frac{1}{1+9x}$ $\qquad\qquad$ $g(x) = \arctan(x/3)$

31. Find the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{3^n\sqrt{n+1}}$ given that the radius of convergence is 3.

32. Evaluate the indefinite interval as an infinite series:

    $\displaystyle\int \sqrt{x^3+1}\,dx$ $\qquad\qquad$ $\displaystyle\int \frac{\sin x}{x}\,dx$

    $\displaystyle\int \frac{t}{1-t^3\,dt}$ $\qquad\qquad$ $\displaystyle\int \frac{\ln(1-t)}{t}\,dt$