Key

20 min.

Quiz 7
Math 252

$\sin 2x = 2\sin x\cos x$ $\qquad$ $\cos 2x = \cos^2 x - \sin^2 x$ $\qquad$ $\sin x\cos y = \frac{1}{2}[\sin(x-y)+\sin(x+y)]$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$ $\qquad$ $\cos^2 x = \frac{1}{2}(1+\cos 2x)$ $\qquad$ $\sin x\sin y = \frac{1}{2}[\cos(x-y)-\cos(x+y)]$

$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$ $\qquad$ $\cos x\cos y = \frac{1}{2}[\cos(x-y)+\cos(x+y)]$

Show *all* your work (algebraically or geometrically) for each and simplify. No credit is given without supporting work.

1. [1] Write out the form of the partial fraction decomposition of the function (as done in class Wednesday). Do *not* determine the numerical values of the coefficients.

§7.4 36

$$\frac{1}{(x^2-9)^2} = \frac{1}{(x+3)^2(x-3)^2}$$

$$\frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x-3} + \frac{D}{(x-3)^2}$$

+½ $\qquad$ +½

2. [3] Find ONLY ONE of the following. Indicate clearly which one you want graded by completely crossing out the problem you do not want graded.

$$\int \cot^3\alpha \csc^3\alpha \, d\alpha$$

§7.2 #39

$\sin^2\theta + \cos^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta$

$\frac{d}{d\theta}(\cot\theta) = -\csc^2\theta$

$\frac{d}{d\theta}(\csc\theta) = -\csc\theta\cot\theta$

tried id +½

to a form that worked +1

notation +½

$= \int \cot^2\alpha\csc^2\alpha \, \cot\alpha\csc\alpha \, d\alpha$

$= \int (\csc^2\alpha - 1)\csc^2\alpha \, \cot\alpha\csc\alpha \, d\alpha$

integrated poly +½

$u = \csc\alpha$

$du = -\csc\alpha\cot\alpha \, d\alpha$

$= \int (u^2-1)u^2(-1)\,du = \int -u^4 + u^2\,du$

$= -\frac{1}{5}u^5 + \frac{1}{3}u^3 + c$

$= -\frac{1}{5}\csc^5\alpha + \frac{1}{3}\csc^3\alpha + c$ +½

$$\int \tan^3 x \, dx$$

§7.2 example 7

tried id +½

to a form that worked +1

notation +½

$\tan^2\theta + 1 = \sec^2\theta$

$= \int \tan^2 x \tan x \, dx = \int (\sec^2 x - 1)\tan x \, dx$

$= \int \tan x \sec^2 x \, dx - \int \tan x \, dx$

$u = \tan x$ $\qquad$ $u = \cos x$

$du = \sec^2 x \, dx$ $\qquad$ $du = -\sin x \, dx$

$= \int u \, du - \int \frac{1}{u}(-1)\,du$

integrated +½

$= \frac{1}{2}u^2 + \ln|u| + c$

$= \frac{1}{2}\tan^2 x + \ln|\cos x| + c$ +½

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

3. [3] Find ONLY ONE of the following. Indicate clearly which one you want graded by completely crossing out the problem you do not want graded.

$$\int \frac{x}{\sqrt{x^2-7}}\,dx \qquad \S 7.3 \#17$$

Substitution

let $u = x^2 - 7$ (+½)

$du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$ (+½)

$= \int \frac{\frac{1}{2}}{\sqrt{u}}\,du = \frac{1}{2}\int u^{-\frac{1}{2}}\,du$ (+1)

$= \frac{1}{\cancel{2}}\cancel{2}\, u^{\frac{1}{2}} + c$ (+½)

$= \sqrt{x^2-7} + c$ (+½)

— or —

trig sub.

$x = \sqrt{7}\sec\theta$ (+½) $\quad dx = \sqrt{7}\sec\theta\tan\theta\,d\theta$ (+½)

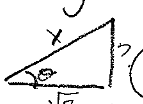
$\int \frac{\sqrt{7}\sec\theta}{\sqrt{7\sec^2\theta - 7}}\sqrt{7}\sec\theta\tan\theta\,d\theta = \int \frac{7\sec^2\theta\,\cancel{\tan\theta}\,d\theta}{\sqrt{7}\cancel{\tan\theta}}$ (+½) (+½)

$= \int \sqrt{7}\sec^2\theta\,d\theta = \sqrt{7}\tan\theta + c$

$?^2 = x^2 - 7$

$= \cancel{\sqrt{7}}\frac{\sqrt{x^2-7}}{\cancel{\sqrt{7}}} + c$

$= \sqrt{x^2-7} + c$



(+½)

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx \qquad \S 7.3 \text{ example } 1$$ (+½)

let $x = 3\sin\theta$ (+½) $\quad dx = 3\cos\theta\,d\theta$

$\int \frac{\sqrt{9 - 9\sin^2\theta}}{9\sin^2\theta}\,3\cos\theta\,d\theta$ (+½) (+½)

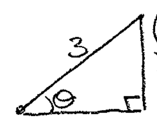
$= \int \frac{\cancel{3}\sqrt{1-\sin^2\theta}}{\cancel{9}\sin^2\theta}\cdot\cancel{3}\cos\theta\,d\theta$

for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$= \int \frac{\cos^2\theta}{\sin^2\theta}\,d\theta$

$= \int \cot^2\theta\,d\theta = \int \csc^2\theta - 1\,d\theta$ (+½)

$= -\cot\theta - \theta + c$



(+½)

$= -\frac{\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + c$

Sohcahtoa

$?^2 = 3^2 - x^2$

4. [3] Use partial fractions to find

$$\int \frac{1}{x^2-1}\,dx = \int \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1}\,dx \qquad \S 7.4 \#11$$

$\frac{A}{x+1} + \frac{B}{x-1} = \frac{1}{x^2-1}$ (+1)

$\Rightarrow \frac{Ax - A + Bx + B}{x^2-1} = \frac{1}{x^2-1}$

$\Rightarrow (A+B)x = 0x$

$-A + B = 1$

$\Rightarrow A = -B$

and $B = \frac{1}{2}$ (+½)

so $A = -\frac{1}{2}$

ck $\frac{-\frac{1}{2}}{x+1} + \frac{+\frac{1}{2}}{x-1} = -\cancel{\tfrac{1}{2}x} + \frac{1}{2} + \cancel{\tfrac{1}{2}x} + \frac{1}{2}$ ✓

$= -\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + c$ (+½) (+½)

notation (+½)