Quiz 6
Math 252

Key

$\sin 2x = 2\sin x\cos x$ $\qquad$ $\cos 2x = \cos^2 x - \sin^2 x$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$ $\qquad$ $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

Show *all* your work (algebraically or geometrically) for each. No credit is given without supporting work.

1. [4] Find ONLY ONE of the following. Indicate clearly which one you want graded by completely crossing out the problem you do not want graded.

§7.2 #15

$$\int \frac{\cos^5\alpha}{\sqrt{\sin\alpha}}\,d\alpha$$

$$\int \frac{\cos^2\alpha\cos^2\alpha\cos\alpha}{\sqrt{\sin\alpha}}\,d\alpha$$

$u=\sin\alpha$

$du=\cos\alpha\,d\alpha$

looking to sub (+½)
sub choice (+½)
sub right (+½)

$$\int \frac{(1-\sin^2\alpha)(1-\sin^2\alpha)\cos\alpha\,d\alpha}{\sqrt{\sin\alpha}}$$

used identities (+½)

$$=\int \frac{(1-u^2)^2}{u^{1/2}}\,du$$

got it to a workable form (+1)

$$=\int \frac{1-2u^2+u^4}{u^{1/2}}\,du$$

$$=\int u^{-1/2}-2u^{3/2}+u^{7/2}\,du$$

(+½)

$$=2u^{1/2}-2\cdot\frac{2}{5}u^{5/2}+\frac{2}{9}u^{9/2}+C$$

$$=2\sqrt{\sin\alpha}-\frac{4}{5}\left(\sqrt{\sin\alpha}\right)^5+\frac{2}{9}\left(\sqrt{\sin\alpha}\right)^9+C$$

(+½)

or

§7.2 #9 or ex 4

$$\int \sin^4 x\,dx$$

used identities (+½)
got it to workable form (+1)

$$\int\left[\frac{1}{2}(1-\cos 2x)\right]^2dx=\frac{1}{4}\int 1-2\cos 2x+\cos^2 2x\,dx$$

$$=\frac{1}{4}\left[\int 1\,dx-2\int\cos 2x\,dx+\int\cos^2 2x\,dx\right]$$

$u=2x$

$du=2dx$

used id/IP in round 2 (+½)
no nonsense (+½)

$$=\frac{1}{4}\left[x-2\int\frac{1}{2}\cos u\,du+\int\frac{1}{2}(1+\cos 2(2x))\,dx\right]$$

$$=\frac{1}{4}\left[x-\sin u+\frac{1}{2}\int 1+\cos 4x\,dx\right]$$

$$=\frac{1}{4}x-\frac{1}{4}\sin 2x+\frac{1}{8}\left[\int 1+\cos 4x\,dx\right]$$

integrated trig alone (+½)
integrated polys (+½)

$w=4x$

$dw=4dx$

$$=\frac{1}{4}x-\frac{1}{4}\sin 2x+\frac{1}{8}\left[\int 1\,dx+\int\frac{1}{4}\cos w\,dw\right]$$

$$=\frac{1}{4}x-\frac{1}{4}\sin 2x+\frac{1}{8}\left[x+\frac{1}{4}\sin w\right]+C$$

$$=\frac{1}{4}x-\frac{1}{4}\sin 2x+\frac{1}{8}x+\frac{1}{32}\sin 4x+C$$

(+½)

IP formula (+1)

2. [6] Find ONLY TWO of the following. Indicate clearly which two you want graded by completely crossing out the problem you do not want graded.

$\int \arctan 4t\,dt$ §7.1 #11

looking for u+dv (+1/2)

$u = \arctan 4t \qquad v = t$

$du = \frac{1}{1+(4t)^2} \cdot 4\,dt \qquad dv = dt$

got one that is reasonable (+1/2)

filled in du + v right (+1)

$= t \cdot \arctan 4t - \int t \cdot 4 \cdot \left(\frac{1}{1+16t^2}\right) dt$

$= t \cdot \arctan 4t - 4 \int \frac{t}{1+16t^2}\,dt$

used formula right (+1/2)

$w = 1+16t^2$

$dw = 32t\,dt$

$\Rightarrow \frac{1}{32}\,dw = t\,dt$

$= t \cdot \arctan 4t - 4 \int \left(\frac{1}{32}\right) \frac{1}{w}\,dw$

$= t \cdot \arctan 4t - \frac{4}{32} \ln|w| + c$

$= t \cdot \arctan 4t - \frac{1}{8} \ln|1+16t^2| + c$

(+1/2)

§7.1 #1 $\int x^2 \ln x\,dx$

looking for u+dv (+1/2)

$\cancel{u = x^2}$

$\cancel{du = 2x\,dx}$

got one that is reasonable (+1/2)

$u = \ln x \qquad v = \frac{1}{3}x^3$

$du = \frac{1}{x}\,dx \qquad dv = x^2\,dx$

filled in du + v right (+1)

$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\,dx$

used formula right (+1/2)

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2\,dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3} \cdot \frac{1}{3}x^3 + c$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$

(+1/2)

§7.1 #5 $\int r e^{\frac{r}{2}}\,dr$

looking for u+dv (+1/2)

got one that is reasonable (+1/2)

$u = r \qquad v = 2e^{\frac{r}{2}}$

$du = dr \qquad dv = e^{\frac{r}{2}}\,dr$

filled in du + v right (+1)

$= r \cdot 2e^{\frac{r}{2}} - \int 2e^{\frac{r}{2}}\,dr$

used formula right (+1/2)

$= 2re^{\frac{r}{2}} - 2\int e^{\frac{r}{2}}\,dr$

$= 2re^{\frac{r}{2}} - 2 \cdot 2\,e^{\frac{r}{2}} + c$

$= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}} + c$

(+1/2)