Quiz 4
Math 252

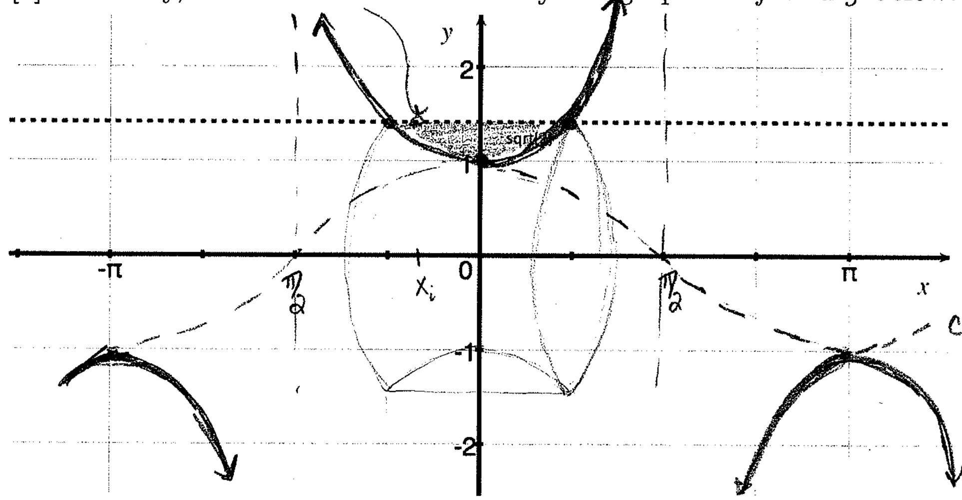
Key

Show *all* your work (algebraically, geometrically, or calculus) for the following. Since the answer is sometimes given to you, it really is the supporting work that is being graded.

1. Let $f(x) = \sec x$ and $g(x) = \sqrt{2}$.

bounded by $x = -\pi$ & $x = \pi$

(a) [1] Carefully, draw the area bounded by the graphs of f and g below.



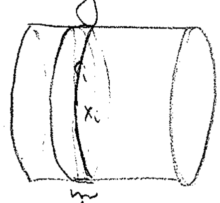
$\sec x = \dfrac{1}{\cos x}$

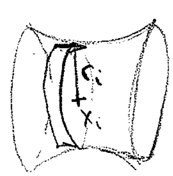
$\sec \frac{\pi}{4} = \dfrac{1}{\cos \frac{\pi}{4}} = \dfrac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

(b) [4] Find the volume of the solid obtained by rotating the region bounded by the graphs of f and g around the x-axis.

broken/sub (+½)
order (+½)
square in right (+1)
bounds (+½)
integrand: ie $\sqrt{2}$ & $\sec x$

Vol of cylinder − Vol of inside $= \displaystyle\int_?^{*} \pi \sqrt{2}^2\,dx - \int_?^{*} \pi \sec^2 x\,dx$





Vol of approx cyl
$(\Delta x)\cdot \pi r_i^2$
$(\Delta x)\cdot \pi \sqrt{2}^2$

Vol of approx cyl
$(\Delta x)\cdot \pi r_i^2$
$\Delta x\, \pi (\sec x_i)^2$

to find the bounds we need to find out when f crosses the line $y = \sqrt{2}$. (+1)

partial. +½ was looking.

So $f(x) = \sqrt{2}$
$\iff \sec x = \sqrt{2}$
$\iff \dfrac{1}{\cos x} = \sqrt{2}$

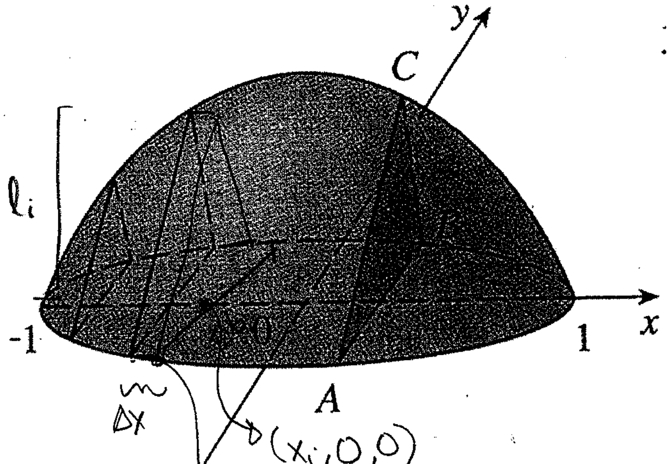
$1 = \sqrt{2}\cos x$
$\dfrac{1}{\sqrt{2}} = \cos x$
$\Rightarrow x = \frac{\pi}{4} + 2\pi k$
$-\frac{\pi}{4} + 2\pi k$.

antider (+½)

So Volume is $\displaystyle\int_{-\pi/4}^{\pi/4} \pi\, 2\,dx - \int_{-\pi/4}^{\pi/4} \pi \sec^2 x\,dx = 2\pi x \Big]_{-\pi/4}^{\pi/4} - \pi \tan x \Big]_{-\pi/4}^{\pi/4}$

$= 2\pi\left[\frac{\pi}{4} + \frac{\pi}{4}\right] - \pi\left[\tan\frac{\pi}{4} - \tan\frac{-\pi}{4}\right] = 2\pi\,\frac{2\pi}{4} - \pi[1 + 1] = \pi^2 - 2\pi$

2. [5] Let X be the solid that was described in Wednesday's lecture and depicted below. That is, X has a circular base of radius 1. Parallel cross sections perpendicular to the base are equilateral triangles. Find the volume of the solid. *Hint: the answer is* $\frac{4\sqrt{3}}{3}$.



Cut // to z-y plane

approx cyl
$\Delta x \cdot \frac{1}{2} b_i l_i$

approx cyc/ create notation (+1)

+1/2 if (height)(area)

need to write b_i & l_i as functions of x. (+1/2)

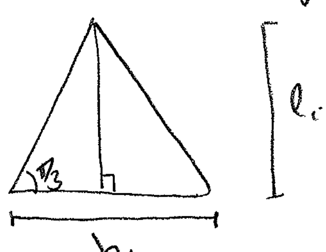
note $x_i^2 + (\frac{1}{2}b_i)^2 = 1$

$\Rightarrow \frac{1}{4}b_i^2 = 1 - x_i^2$

$b_i = \sqrt{4 - 4x_i^2}$

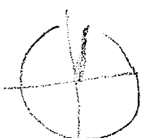
$b_i = 2\sqrt{1 - x_i^2}$ (+1)

+1/2 if looking



Sohcahtoa

$\tan \pi/3 = \frac{l_i}{(\frac{1}{2}b_i)}$



$\Rightarrow \frac{1}{2}b_i \tan \pi/3 = l_i$

+1/2 if looking

$\Rightarrow \frac{1}{2} 2\sqrt{1-x_i^2} \cdot \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = l_i$

$\Rightarrow l_i = \sqrt{3}\sqrt{1-x_i^2}$ (+1)

So approx cyc

$(\Delta x)(\cancel{\tfrac{1}{2}})(\cancel{2}\sqrt{1-x_i^2})(\sqrt{3}\sqrt{1-x_i^2})$

$(1-x_i^2)\ \sqrt{3}\ (\Delta x)$

notation (+1/2)

(+1)

$\Rightarrow \text{Vol} = \int_{-1}^{1} \sqrt{3}(1-x^2)\,dx$

$= \sqrt{3}\int_{-1}^{1}(1-x^2)\,dx$

$= \sqrt{3}\left[x - \frac{1}{3}x^3\right]_{-1}^{1}$

$= \sqrt{3}\left[(1-\frac{1}{3}) - (-1+\frac{1}{3})\right]$

$= \sqrt{3}\left[\frac{2}{3} + 1 - \frac{1}{3}\right] = \sqrt{3}\left[1 + \frac{1}{3}\right]$

$= \frac{4\sqrt{3}}{3}$