give domain.

Quiz 3
Math 252

Name: Key

Show *all* your work (algebraically or geometrically) for each and simplify. No credit is given without supporting work.

1. [2] Evaluate the following *or* explain why your theorems don't apply.

$\int_0^{\frac{\pi}{4}} \sec^2 t\, dt$ (+1/2 thm applies)

$\tan t \Big]_0^{\pi/4} = \tan \frac{\pi}{4} - \tan 0$ (+1/2 antideriv)

$= 1 - 0 = 1$ (+1/2 correct)

Ck $\frac{d}{dt}(\tan t) = \frac{d}{dt}\left(\frac{\sin t}{\cos t}\right) = \frac{d}{dt}(\sin t (\cos t)^{-1})$

$= \frac{-\sin t}{\cos^2 t}(-\sin t) + \frac{\cos t}{\cos t}$

$= \frac{\sin^2 t}{\cos^2 t} + 1 = \tan^2 t + 1 = \sec^2 t$ ✓

* $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \tan^2 t + 1 = \sec^2\theta$

$\int_{-2}^{1} x^{-4}\, dx$

$x^{-4} = \frac{1}{x^4}$ is not cont (+1/2)

on the interval $[-2, 1]$

in fact the graph suggests it should be a positive area

but if you use $-\frac{1}{3}x^{-3}\Big]_{-2}^{1}$ you'd get a neg answer.

2. [2] Find the following *or* explain why your theorems don't apply.

$\int \frac{1}{x^2}\, dx = \int x^{-2}\, dx$ (+1/2)

$= -x^{-1} + C$ (+1/2)

Ck: $\frac{d}{dx}\left(\frac{-1}{x}\right) = \frac{d}{dx}(-x^{-1})$

$= x^{-2}$ ✓

$\int e^{5x}\, dx = \int e^u \frac{1}{5}\, du$

let $u = 5x$

$du = 5dx$

$\Rightarrow \frac{1}{5}du = dx$

substitution

Lost keeping track of $\frac{1}{5}$ pattern

So $\int e^{5x} dx = \frac{1}{5}\int e^u du$

$= \frac{1}{5}e^u + C$

$= \frac{1}{5}e^{5x} + C$ (+1/2) (+1/2)

full credit even if didn't use sub

Ck: $\frac{d}{dx}\left(\frac{1}{5}e^{5x} + C\right) = e^{5x}$ ✓

3. [2] If $f(1) = 12$, f' is continuous, and $\int_1^4 f'(x)\,dx = 17$, what is the value of $f(4)$?

by FTC II (pretzel +½) $\int_1^4 f'(x)dx = f(4) - f(1)$ +1 b/c $\int_1^4 f'(x)dx = 17$

So $17 = f(4) - f(1)$ b/c $f(1) = 12$,

$\Rightarrow 17 = f(4) - 12$

$\Rightarrow f(4) = 17 + 12$

$f(4) = 29$

4. A particle moves along a line so that its velocity as time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

   (a) [2] Set up but *do not calculate* the definite integral(s) used to find the net displacement of the particle during the time period $1 \le t \le 4$.

$\int_1^4 t^2 - t - 6\,dt$

+½ +1 +½

   (b) [2] Set up but *do not calculate* the the definite integrals used to find the total distance traveled during the time period $1 \le t \le 4$.

$t^2 - t - 6 = (t-3)(t+2)$

$-\left(\int_1^3 t^2 - t - 6\,dt\right) + \int_3^4 t^2 - t - 6\,dt$

bracket each (+½)

broken up (+½)

neg sign (+1)