

Quiz 2

Math 252

Name: Key

Show *all* your work (algebraically or geometrically) for each and simplify. No credit is given without supporting work.

1. [2] Given the graph of f passes through the point $(1, 6)$ and that the slope of its tangent line at $(x, f(x))$ is $2x + 1$, find $f(2)$.

The graph of f passed through $(1, 6) \Rightarrow f(1) = 6$

The slope of its tangent line @ $(x, f(x))$ is $2x + 1 \Rightarrow f'(x) = 2x + 1$

find $f(2)$

since $f'(x) = 2x + 1$

~~not correct~~ $f(x) = x^2 + x + C$ for some C

~~not correct~~ $\frac{d}{dx}(x^2 + x + C) = 2x + 1$ ✓

since $f(1) = 6 = 1^2 + 1 + C \Rightarrow 6 = 2 + C$
 $\Rightarrow C = 4$

so $f(x) = x^2 + x + 4$

$\Rightarrow f(2) = 2^2 + 2 + 4 = 4 + 2 + 4 = 10$

2. [2] Find $\frac{d}{dx} \int_1^{x^4} \sec t \, dt$.

FTC Part 1 & chain rule let $u = x^4$

$\frac{d}{du} \left[\int_1^u \sec t \, dt \right] \cdot \frac{du}{dx} = \sec u \cdot \frac{du}{dx}$

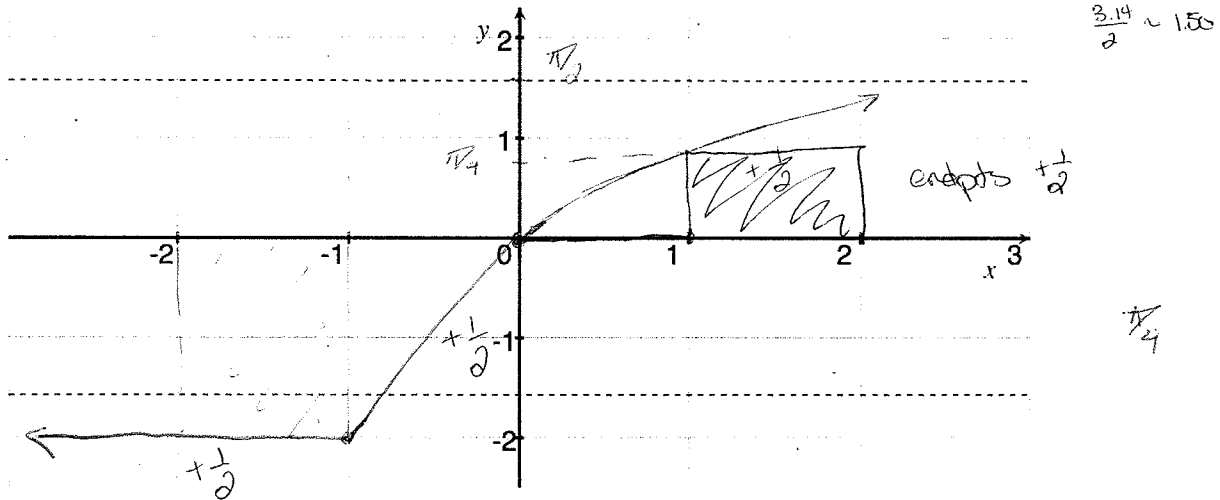
$= (\sec x^4) \cdot 4x^3$

$= 4x^3 \sec x^4$

chain rule

3. [2] Let $f(x) = \begin{cases} -2 & \text{if } x \leq -1 \\ 2x & \text{if } -1 < x \leq 0 \\ \arctan x & 0 < x \end{cases}$

Carefully draw the graph of f on the graph provided below and use this graph to answer the following questions.



(a) [2] Approximate $\int_0^2 f(t) dt$, using two approximating rectangles and left endpoints.
 (Note: you can write this approximate exactly by using the definition of f)

+1 if see boxes $1 \cdot f(0) + 1 \cdot f(1)$
 $1 \cdot 0 + 1 \cdot \frac{\pi}{4} = \frac{\pi}{4}$
 $\frac{1}{2} \quad \frac{1}{2}$

$\tan^{-1} 0 = 0$
 $\tan^{-1} 1 = \frac{\pi}{4}$

(b) [2] Evaluate $\int_{-2}^0 f(t) dt$ exactly.

$-1 \cdot 2 + \frac{1}{2}(1)(2) = -2 + 1 = -1$
 $\frac{1}{2}$ if see shaded
 $\frac{1}{2}$ if sign
 $+1$ if 3