MIDTERM 2 # Math 252 PRACTICE

NAME:

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

   T F If $f$ is differentiable, and $f'(c) = 0$, then $f(c)$ is a local maximum.

   T F Substitution yeilds: $\int_0^1 y(y^2+1)^5 dy = \int_0^1 \frac{1}{2}u^5 du$

   T F $\int_{-1}^1 \frac{1}{x^2}dx = \frac{-1}{x}\Big|_{-1}^1 = \frac{-1}{1} - \frac{-1}{-1} = -2$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [5]Given the graph of a force function with respect to distance below, graph the total work as a function of distance.

3. [10] Let $f(t)$ =??? (pick a function from §6.5 that involves trig). Find the average value of $f$ on the interval $[?, ?]$.

4. (a) [8] Interpret $\int_1^{e^2} \ln x - (\frac{-1}{e(e-1)}x + \frac{e}{e-1})dx$ as the area of a region, by sketching a graph. *Hint: $x = e$ and $x = e^2$ are good points to plot.*

(b) [4] Interpret $\int_1^{e^2} |\ln x - (\frac{-1}{e(e-1)}x + \frac{e}{e-1})|dx$ as the area of a region.

(c) [5] Explain how to evaluate $\int_1^{e^2} |\ln x - (\frac{-1}{e(e-1)}x + \frac{e}{e-1})|dx$, but do *not* perform the evaluation.

5. [15] Consider the solid whose base is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$. The cross-sections perpendicular to the $x$-axis are squares with one side lying along the base. Sketch the volume and then find its volume.

6. [10] Recall the Mean Value Theorem from first term calculus:

   If $g$ is a continuous function on the closed interval $[a, b]$, and differentiable on the open interval $(a, b)$, then there is a number $c$ between $a$ and $b$ such that

$$g'(c) = \frac{g(b) - g(a)}{b - a}$$

   Prove that if $f$ is continuous on $[a, b]$, then there exists a number $c$ between $a$ and $b]$ such that

$$\int_a^b f(x)dx = f(c)(b - a)$$

   .

   *Hint: consider* $F(t) = \int_a^t f(x)dx.$