MIDTERM 1 # Math 252 PRACTICE

NAME:

1. [3] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let a, b, and c be constants. Assume f and g are continuous.

   T F $\int_a^b cf(x)dx = c\int_a^b f(x)\,dx$

   T F $\int f(x)g(x)dx = \int f(x)\,dx \int g(x)dx$

   T F $\int_{-1}^{1} \frac{1}{x^2}\,dx = \frac{-1}{x}\Big|_{-1}^{1} = \frac{-1}{1} - \frac{-1}{-1} = -2$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [4] *Carefully* write down the second part of the Fundamental Theorem of Calculus.

3. [4] Find the equation of the line that is tangent to the graph of $y = \ln x$ at $x = e^a$ for some constant a.

$$f(x) = \begin{cases} -\sqrt{4-x^2};\ \text{if } -2 \le x \le 2 \\ x-2;\ \text{if } 2 < x \end{cases}$$

4. Refer to the above definition of $f(x)$ to answer the following questions.

   (a) [2] Carefully graph $f(x)$ on the set of axis provided.

   (b) [3] Use your above graph to find $\int_{-2}^{4} f(x)\,dx$.

   (c) [4] Give a rough sketch the graph of $\int_{-2}^{x} f(t)\,dt$ above and clearly mark it as such.

5. [3] Find $\frac{d}{dx}\int_0^{\tan x}\sqrt{1+r^3}\,dr$. You need not simplify.

6. [5] Set up the definite integral that gives the area of the region bounded by $y+1=x$ and the parabola $\frac{1}{2}y^2-3=x$. Do *not* evaluate the integral.

7. [6 each] Evaluate *ONLY TWO* of the following. Indicate clearly which two you want graded by completely striking the problem you do not want graded.

(a) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \sin x}{1+x^6}\, dx$

(b) $\displaystyle\int x^3\sqrt{x^2+1}\, dx$

(c) $\displaystyle\int \frac{e^x}{1+e^{2x}}\, dx$

8. [10] Kobayashi has won the hot dog-eating world championship six times. Recently he challenged a giant bear to a 3 minute hot dog-eating contest. Kobayashi found that the rate he can eat hot dogs goes down as time goes by and can be modeled by $k(t) = \frac{12}{(t+1)^2} + 24$, were t is measured in minutes. The bear isn't quite as used to the system and seems to start with a slower rate that gets larger and is well modeled by $b(t) = 8t^3 + 20$. Find out how many hot dogs Kobayashi and the bear eat and determine who won the context.