FINAL **Math 252** PRACTICE

NAME:

1. [10] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let a and b be constants with $a \leq b$ and $f(x)$ and $g(x)$ be continuous functions on $[a, b]$.

   T F $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

   T F $\displaystyle\int_a^b f(x)g(x)\,dx = \int_a^b f(x)\,dx * g(x) + f(x) * \int_a^b g(x)\,dx$

   T F $\frac{\alpha x^3+\beta x^2+\gamma}{x(x-1)^2(x^2+1)^3}$ can be put in the form of $\frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1}$ for any constants α, β, and γ.

   T F If f is continuous, then $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = \lim_{t\to\infty}\int_{-t}^{t} f(x)\,dx$.

   T F If $\int_a^\infty f(x)\,dx$ and $\int_a^\infty g(x)\,dx$ are both convergent, then $\int_a^\infty f(x) + g(x)\,dx$ is convergent.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [2] *Carefully* write down the first Fundamental Theorem of Calculus.

3. [2] *Carefully* write down the second Fundamental Theorem of Calculus.

4. [5] $\dfrac{d}{dt}\displaystyle\int_0^t e^{x^2}\,dx$ $\qquad\qquad$ $\dfrac{d}{dx}\displaystyle\int_0^{x^2+3x} e^{t^2}\,dt$

5. [5] The graphs of $JIM(t)$ and $JOHN(t)$ below trace the velocity of Jim and John respectively from time 0, measured in minutes. Explain what the physical meaning of $\int_0^5 JIM(t) - JOHN(t)\,dt$ is.

6. [5] Let $F(x) = \int_0^x f(t)\,dt$ and $f(t)$ have the graph given below. Sketch the graph of $F(x)$.

7. Let $f(x) = \ln x$.

   (a) [5] Find the average value of f on the interval $[1, e]$.

   (b) [5] Is there a number c between 1 and e so that $f(c)$ is equal to the value you found in part a? Explain, *briefly* why or why not.

8. [10 *each*] Evaluate the following if they exist.

(a) $\displaystyle\int_0^{\frac{\pi}{4}} \sec^4 x \tan^4 x \, dx$

(b) $\displaystyle\int x \cos^2 x \, dx$

(c) $\displaystyle\int_1^{\infty} \frac{1}{x^2} \, dx$

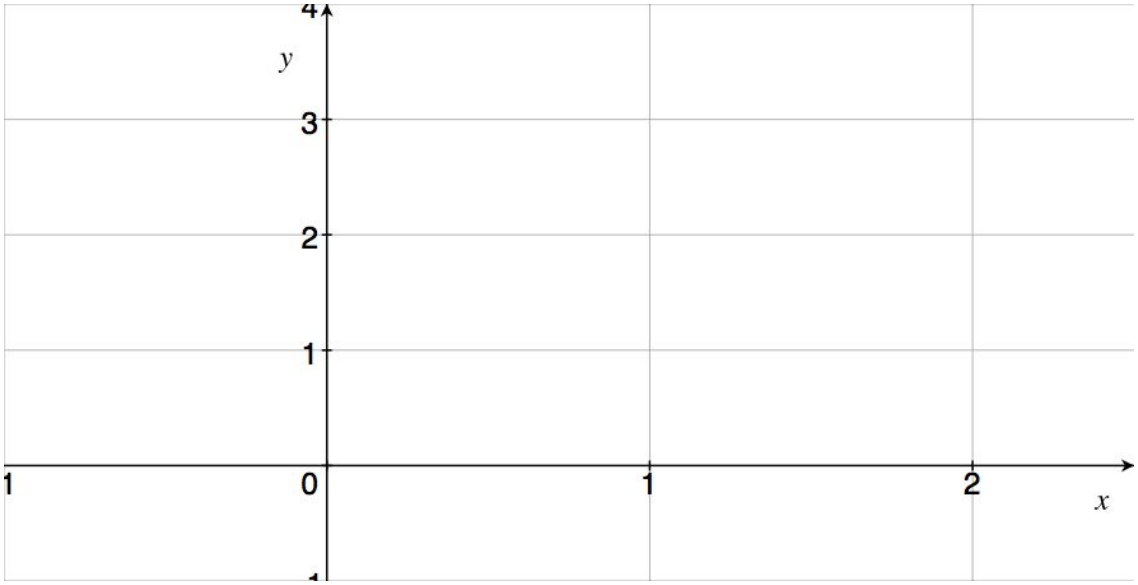
(d) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$

(e) $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$

(f) $\displaystyle\int \frac{17x-1}{2x^2+3x-2}\,dx$

9. Let $f(x) = x^2$.

(a) [3] Carefully draw the graph of f on the set of axis provided.



(b) [4] Let g be the function tangent to f at $x = 1$. Find the rule for g and draw the graph of g on the above graph.

(c) [6] Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the x-axis.

10. [10] Use calculus to show $\frac{L^2 h}{3}$ is the volume of a pyramid whose base is a square with side L and whose height is h.

11. [10] A tank has the shape of an inverted circular cone with height 10m and base 4 m. It is filled with water to a height of 8m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000kg/m^3.)