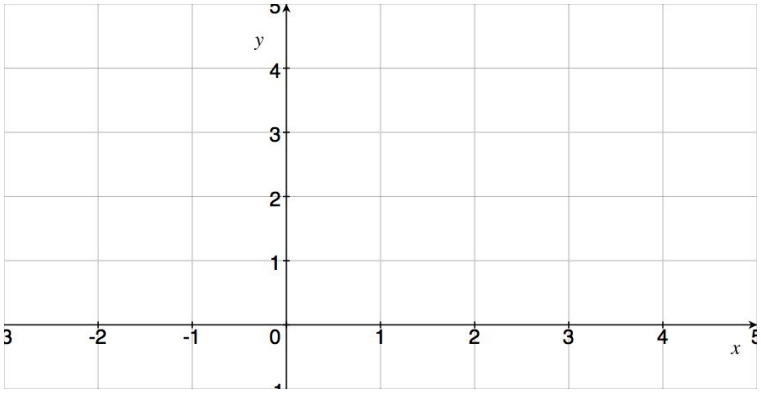
MIDTERM 2 **Math 252** Spring 2009

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

1. 

   (a) [3] Interpret $\int_0^4 \sqrt{x+2} - x\,dx$ as an area of a region.

   (b) [3] Interpret $\int_0^4 |\sqrt{x+2} - x|\,dx$ as an area of a region.

2. (a) [3] Write down the Meal Value Theorem for Integrals.

   (b) [2] What is the geometric interpretation?

3. A reasonable rule to describe the force required to maintain a spring stretched x units beyond its natural length is given by Hooke's Law: $F(x) = kx$.

   (a) [2] A spring has a natural length of 20m (yes, it is a very large spring). If a 25-N force is required to keep it stretched to a length of 30 m, find the rule for the force function.

   (b) [4] How much work is required to stretch it from 20 m to 25m?

4. [4] Find the average value of the function $h(x) = \cos^4 x \sin x$ on the interval $[0, \pi]$.

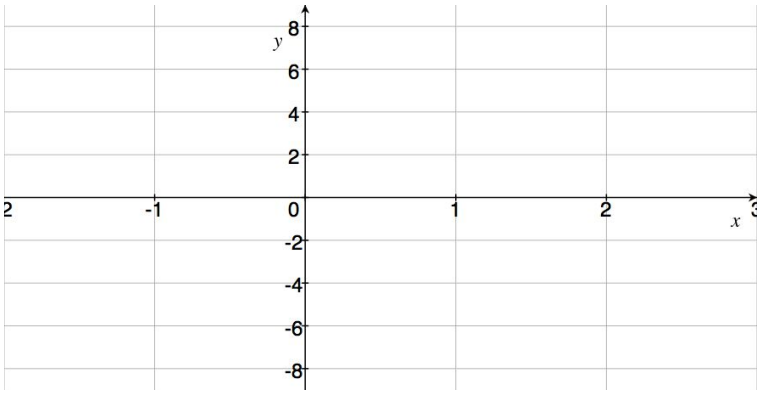
5. Let $f(x) = x^2$ and $g(x) = 2x - 1$.

(a) [3] Carefully draw the graph of f and g on the set of axis provided. It can be shown that g is the line tangent to the graph of f when $x = 1$.

(b) [6] Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the x-axis.

6. [7] Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the x-axis. Be clear about what methods you use

   (Are you using disks or cylindrical shells? Are you integrating with respect to x or y?)



7. [8] Use calculus to show the volume of a pyramid whose base is a square with side L and whose height is h is $\dfrac{L^2 h}{3}$.

8. Recall the Fundamental Theorem of Calculus Part 2: If f is continuous on $[a, b]$, then

$$\int_a^b f(x)/, dx = F(b) - F(a)$$

where F is any antiderivative of f. Follow the steps below to show that *any* antiderivative will work.

(a) [1] Let $g(x) = \int_a^x f(t)\, dt$. What is $g'(x)$?

(b) [1] Let F be an antiderivative to f. What is $F'(x)$?

(c) [3] You know from Math 251 that if $F'(x) = g'(x)$ there exists a constant c so that $F(x) = g(x) + c$. Use this to show $F(b) - F(a) = \int_a^b f(x)\, dx$.