MIDTERM 1 # Math 252 Spring 2009

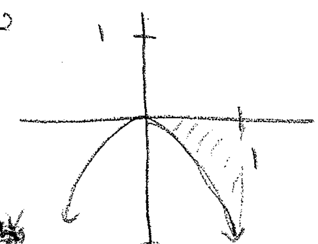
NAME: Key

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

1. [4] Explain in your own words what the following mean.

   (a) $\int_0^1 -x^2\,dx = \frac{-1}{3}$

The area bounded by the x-axis & the graph of $-x^2$ from $x=0$ to $x=1$ is $\frac{1}{3}$ units^2 and below the x-axis.



no bounded/between $-\frac{1}{2}$

- or -

$$\lim_{n\to\infty}\left(\sum_{i=1}^{3} -(x_i^*)^2 \Delta x\right)$$

left & right side (+1/3)

bottom (+1/3)

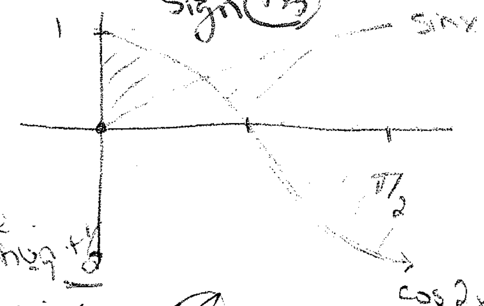
sign (+1/3)

area (+1/3)

   (b) $\int_0^{\frac{\pi}{2}} |\sin x - \cos 2x|\,dx = \frac{3}{2}\sqrt{3} - 1$

§6.1 #

The area bounded between the graphs of $\sin x$ & $\cos 2x$ from $x=0$ to $x=\frac{\pi}{2}$.



Partial picture functions +1/2

left & right (+1/2)

area (+1/2)

top (+1/2)

bottom (+1/2)

2. [4] *Carefully* write down the First Fundamental Theorem of Calculus.

Let f be a con't function on $[a,b]$ (+1)

cont (+1) I.V./Riem

Then the function $\int_a^x f(t)\,dt$ for $a \le x \le b$ (+1) defined right.

is con't on $[a,b]$ and differentiable on (a,b), and

$$\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x).$$

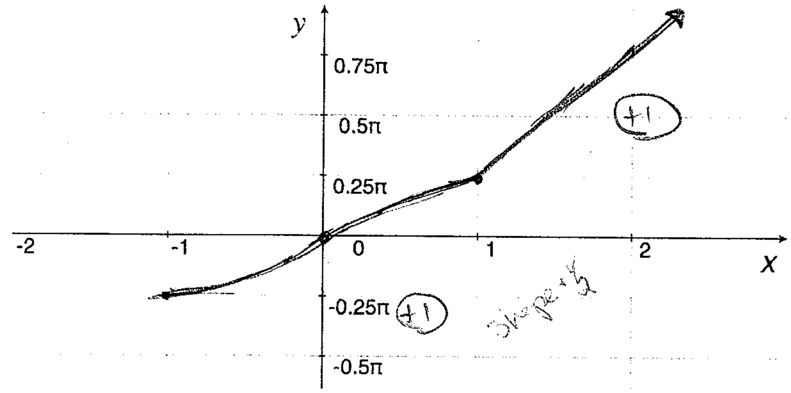
(+1) looking for this.

$$f(x) = \begin{cases} \arctan x & \text{if } -1 \le x \le 1, \\ \frac{\pi}{2}x - \frac{\pi}{4} & \text{if } 1 < x \end{cases}$$

3. Refer to the above definition of $f(x)$ to answer the following questions.

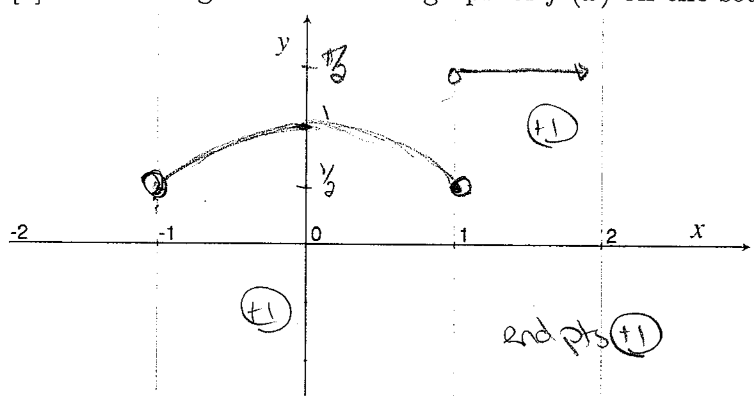
(a) [2] Carefully graph $f(x)$ on the set of axis provided.

prereq math 211 + 112



(b) [3] Give a rough sketch of the graph of $f'(x)$ on the set of axis provided.

prereq math 251



note $\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$

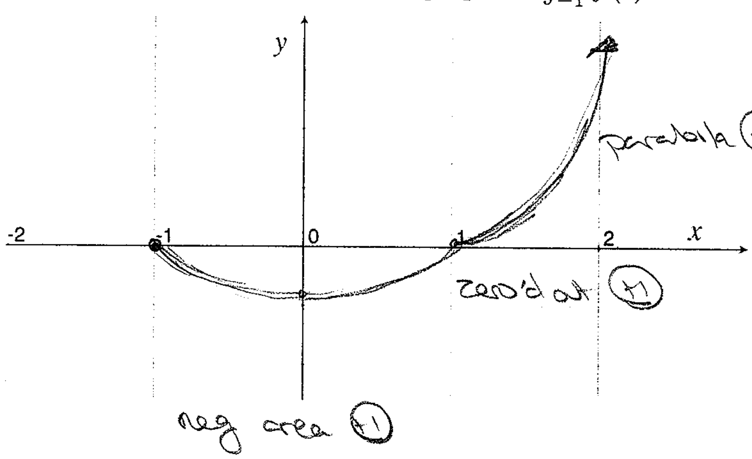
$\frac{d}{dx}\left(\frac{\pi}{2}x - \frac{\pi}{4}\right) = \frac{\pi}{2}$

$f'(-1) = \frac{1}{1+(-1)^2} = \frac{1}{1+1} = \frac{1}{2}$

$f'(0) = \frac{1}{1+0^2} = 1$

(c) [4] Give a rough sketch of the graph of $\int_{-1}^{x} f(t)dt$ on the set of axis provided.

§5.3 #3d
#4e



cont (+1)

4. [4] Given $\int_0^9 f(x)\,dx = 37$ and $\int_0^9 g(x)\,dx = 16$ find the following:

§5.2 #49

(a) $\int_0^9 2f(x) + 3g(x)\,dx$

$$= 2\int_0^9 f(x)\,dx + 3\int_0^9 g(x)\,dx$$

$$= 2 \cdot 37 + 3 \cdot 16 = 74 + 48 = 122$$

Const pull past (+1)
dist over + (+1/2)
notation (+1/2)

(b) $\int_0^3 x g(x^2)\,dx$

§5.5 #82

Let $u = x^2$
$du = 2x\,dx$
$\Rightarrow \frac{1}{2}\,du = x\,dx$

View sub (+1)

$$\int_0^3 x g(x^2)\,dx = \int_0^9 \frac{1}{2} g(u)\,du$$

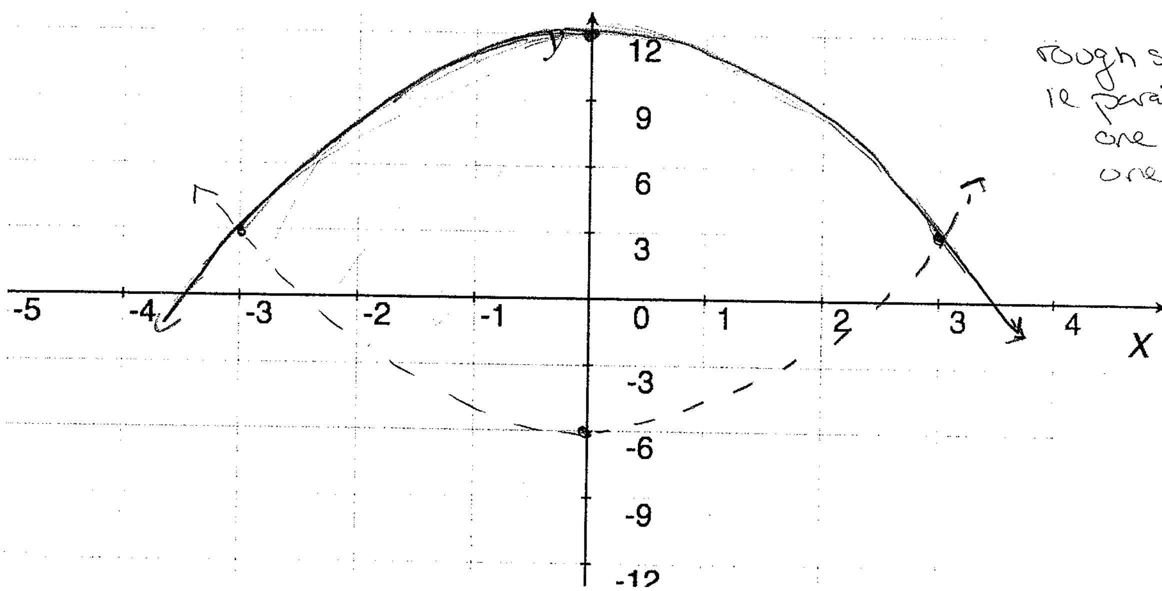
$$= \frac{1}{2}\int_0^9 g(u)\,du$$

$$= \frac{1}{2} \cdot 16 = 8$$

got it (+1/2)
subed right (+1/2)

5. [7] Sketch the region enclosed by the curves $y = 12 - x^2$ and $y = x^2 - 6$ on the set of axis provided. Decide whether to integrate with respect to x or y. Then find the area of the region.

§6.1 #13



rough sketch
ie parabolas.
one up
one down (+1)

$$\int_{-3}^{3} (12-x^2) - (x^2-6)\,dx = \int_{-3}^{3} 18 - 2x^2\,dx = 2\int_{-3}^{3} 9 - x^2\,dx$$

$$= 2\left(9x - \frac{1}{3}x^3\right)\Big|_{-3}^{3}$$

$$= 2[(27-9) - (-27+9)]$$

$$= 2[18 - (-18)]$$

$$= 2(18+18)$$

$$= 2(36) = 72$$

ends (+1)
order of integrand (+1)
paranth. (+1)

Computation +3
try FTC II (+1)
got antider (+1)
(each piece 1/2)
pluged in right order (+1/2)
got it (+1/2)

6. [6 each] Evaluate *ONLY TWO* of the following. Indicate clearly which two you want graded by completely striking the problem you do not want graded.

§5.4 #42

(a) $\int \frac{(x-1)^3}{x^2}\,dx = \int \frac{1x^3 - 3x^2 + 3x - 1}{x^2}\,dx$ notation (+1)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$= \int x - 3 + \frac{3}{x} - \frac{1}{x^2}\,dx$

tried alg (+1)

$= \underbrace{\frac{1}{2}x^2 - 3x}_{\text{poly's (+1)}} + \underbrace{3\ln|x|}_{(+1)} + \underbrace{\frac{1}{x}}_{\text{negpoly (+1)}} + c$ (+1)

Ch 5 review #35

(b) $\int \frac{\sec\theta\tan\theta}{1+\sec\theta}\,d\theta$ notation (+1)

~~let $u = \sec\theta$~~

~~$\frac{du}{d\theta} = \frac{d}{d\theta}(\sec\theta) = \frac{d}{d\theta}\left(\frac{1}{\cos\theta}\right) = \frac{-1}{\cos^2\theta}\cdot -\sin\theta = \frac{\sin\theta}{\cos^2\theta}$~~

~~$= \tan\theta\cdot\sec\theta$~~

~~$\Rightarrow du = (\tan\theta\cdot\sec\theta)\,d\theta$~~

~~$= \int \frac{1}{1+u}\,du$~~

let $u = 1 + \sec\theta$

$\Rightarrow du = \tan\sec\theta\,d\theta$

tried sub (+1)

subed right (+1)

$\int \frac{\sec\theta\tan\theta}{1+\sec\theta}\,d\theta = \int \frac{1}{u}\,du$ (+1)

$= \ln|u| + c$

$= \ln|1+\sec\theta| + c$ (+1)

put back in θ (+1)

§5.5 #27

(c) $\int \frac{z^2}{\sqrt[3]{1+z^3}}\,dz$ notation (+1)

let $u = 1 + z^3$

$du = 3z^2\,dz$

$\Rightarrow \frac{1}{3}du = z^2\,dz$

tried sub (+1)

subed right (+1)

$\int \frac{z^2}{\sqrt[3]{1+z^3}}\,dz = \frac{1}{3}\int \frac{1}{u^{\frac{1}{3}}}\,du$

$= \frac{1}{3}\int u^{-\frac{1}{3}}\,du$ (+1)

$= \frac{1}{\cancel{3}}\,\frac{\cancel{3}}{2}\,u^{\frac{2}{3}} + c$

$= \frac{1}{2}(1+z^3)^{\frac{2}{3}} + c$ (+1)

put back in z (+1)

7. [10] The velocity function (in meters per second) is given for a particle moving along a line by the function $v(t) = 3t - 5$ for $0 \le t \le 3$.

§5.4 #57

(a) Find the net distance traveled by the particle in the given time.

net dist

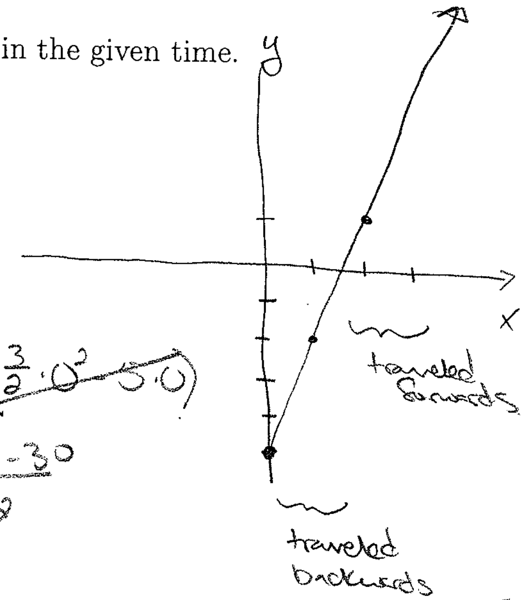
$$= \underbrace{\int_0^3}_{(+1/2)} \underbrace{3t-5\,dt}_{(+1/2)} = \underbrace{3 \cdot \tfrac{1}{2}t^2 - 5t}_{(+1)} \Big]_0^3$$

$$= \left(\tfrac{3}{2}(3)^2 - 5(3)\right) - \left(\tfrac{3}{2}\cdot 0^2 - \cancel{5 \cdot 0}\right)$$

partial graph $+\frac{1}{2}$

$$= \frac{27}{2} - 15 = \frac{27-30}{2}$$

$$= \frac{-3}{2} \quad \text{eval } (+1)$$



note: net the partical moved back $\frac{3}{2}$ m.

(b) Find the total distance traveled by the particle in the given time.

Knew to break up (+1)

broke up in right spot (+1)

found pt to break up (+1)

$0 = 3t - 5$

$\frac{5}{3} = t$

$$\underbrace{\left|\int_0^{5/3} 3t-5\,dt\right|}_{\text{force neg dis. to be recorded as just dis.}} + \int_{5/3}^3 3t-5\,dt$$

$$= -\int_0^{5/3} 3t-5\,dt + \int_{5/3}^3 3t-5\,dt = \overbrace{-\left(\tfrac{3}{2}t^2 - 5t\right)\Big|_0^{5/3}}^{(+1)} + \left(\tfrac{3}{2}t^2 - 5t\right)\Big|_{5/3}^{3}$$

FTC II (+1)

$$= -\left[\left(\tfrac{3}{2}\left(\tfrac{5}{3}\right)^2 - 5\left(\tfrac{5}{3}\right)\right) - \cancel{\left(\tfrac{3}{2}0^2 - 5 \cdot 0\right)}\right] + \left[\left(\tfrac{3}{2}(3)^2 - 5(3)\right) - \left(\tfrac{3}{2}\left(\tfrac{5}{3}\right)^2 - 5\left(\tfrac{5}{3}\right)\right)\right]$$

$$= -\left[\frac{3 \cdot 25}{2 \cdot 9} - \frac{25}{3}\right] + \left[\left(\frac{27}{2} - 15\right) - \left(\frac{3 \cdot 25}{2 \cdot 9} - \frac{25}{3}\right)\right] \quad \text{precalc } (+1)$$

$$= -\left[\frac{25}{6} - \frac{50}{6}\right] + \left[\left(\frac{27-30}{2}\right) - \left(\frac{25}{6} - \frac{50}{6}\right)\right]$$

$$= \frac{25}{6} + \left[\frac{-3}{2} + \frac{25}{6}\right] = \frac{25 - 9 + 25}{6} = \frac{41}{6} \text{ m}$$