Math 252

NAME:

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

1. [4] Explain in your own words what the following mean.

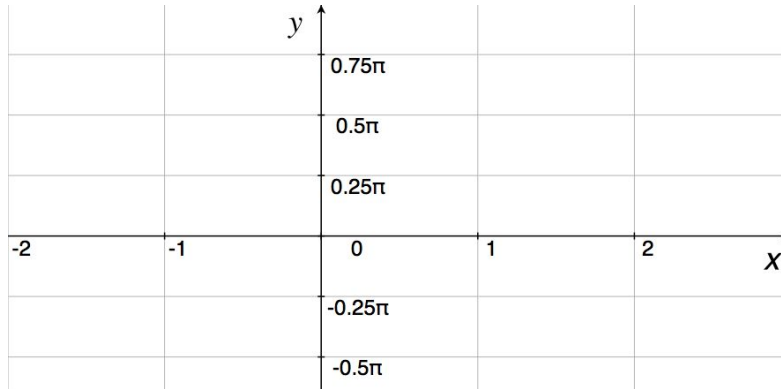   (a) $\int_0^1 -x^2 \, dx = \frac{-1}{3}$

    (b) $\int_0^{\frac{\pi}{2}} |\sin x - \cos 2x| \, dx = \frac{3}{2}\sqrt{3} - 1$

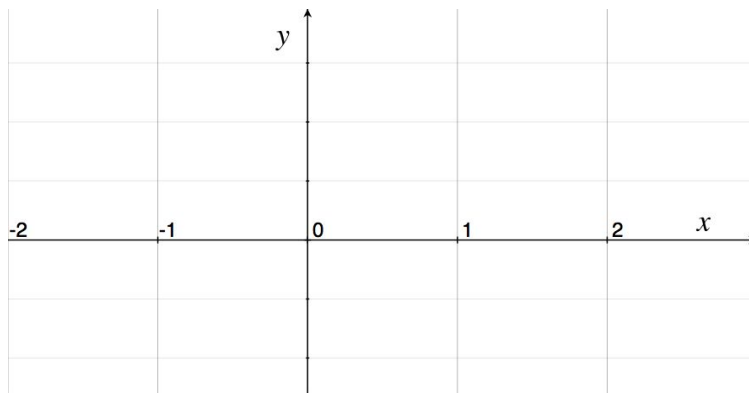2. [4] *Carefully* write down the First Fundamental Theorem of Calculus.

$$f(x) = \begin{cases} \arctan x & \text{if } -1 \le x \le 1, \\ \frac{\pi}{2}x - \frac{\pi}{4} & \text{if } 1 < x \end{cases}$$

3. Refer to the above definition of $f(x)$ to answer the following questions.
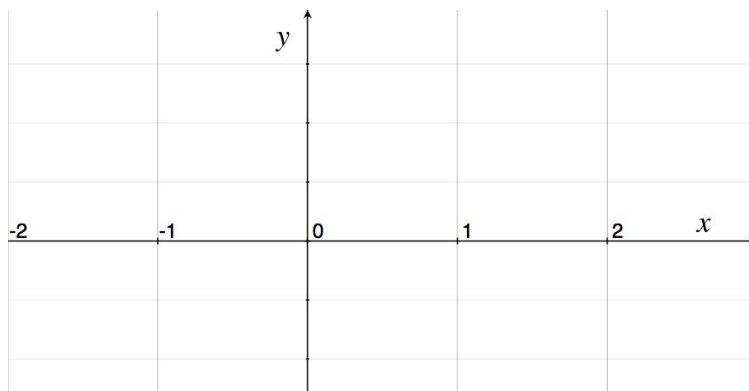
   (a) [2] Carefully graph $f(x)$ on the set of axis provided.

   (b) [3] Give a rough sketch of the graph of $f'(x)$ on the set of axis provided.

   (c) [4] Give a rough sketch of the graph of $\int_{-1}^{x} f(t)dt$ on the set of axis provided.
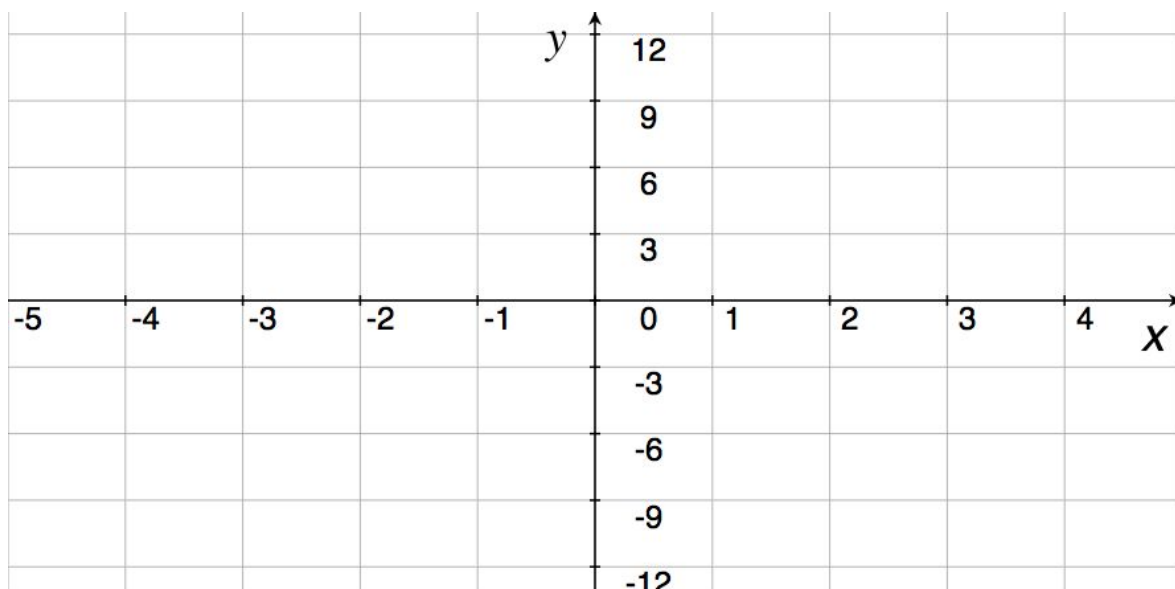
4. [4] Given $\int_0^9 f(x)\,dx = 37$ and $\int_0^9 g(x)\,dx = 16$ find the following:

(a) $\int_0^9 2f(x) + 3g(x)\,dx$

(b) $\int_0^3 xg(x^2)\,dx$

5. [7] Sketch the region enclosed by the curves $y = 12 - x^2$ and $y = x^2 - 6$ on the set of axis provided. Decide whether to integrate with respect to x or y. Then find the area of the region.

6. [6 each] Evaluate *ONLY TWO* of the following. Indicate clearly which two you want graded by completely striking the problem you do not want graded.

(a) $\displaystyle \int \frac{(x-1)^3}{x^2}\,dx$

(b) $\displaystyle \int \frac{\sec\theta\tan\theta}{1+\sec\theta}\,dx$

(c) $\displaystyle \int \frac{z^2}{\sqrt[3]{1+z^3}}\,dx$

7. [10] The velocity function (in meters per second) is given for a particle moving along a line by the function $v(t) = 3t - 5$ for $0 \le t \le 3$.

   (a) Find the net distance traveled by the particle in the given time.

   (b) Find the total distance traveled by the particle in the given time.