MIDTERM 2 # Math 252 Fall 2007

#4

NAME: KEY

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

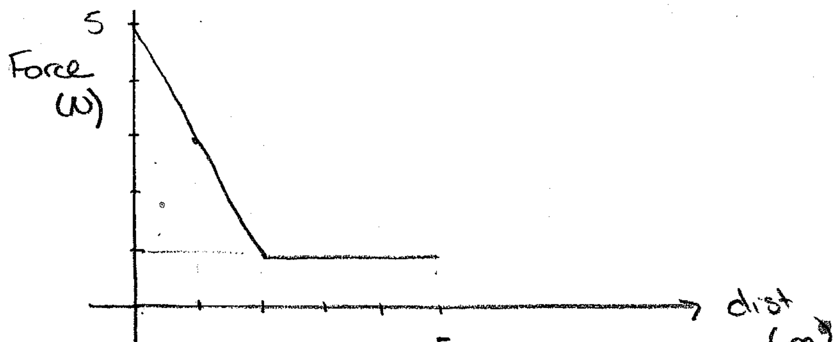
   T F If f is differentiable, and $f'(c) = 0$, then $f(c)$ is a local maximum.

   T F Substitution yeilds: $\int_0^1 y(y^2+1)^5 dy = \int_0^1 \frac{1}{2}u^5 du$

   T F $\int_{-1}^{1} \frac{1}{x^2} dx = \frac{-1}{x}\Big|_{-1}^{1} = \frac{-1}{1} - \frac{-1}{-1} = -2$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [5]Given the graph of a force function with respect to distance below, graph the total work as a function of distance.



$(4\cdot 2)\frac{1}{2} = 4$
$+2$
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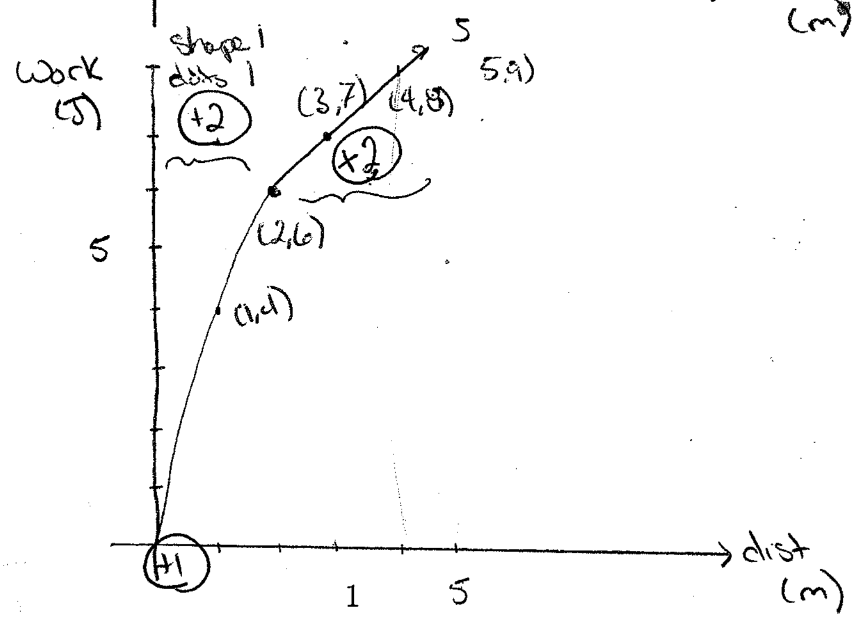
$3 + \frac{1}{2}(2)(1) = 4$



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3. [10] Let $f(t) = t\sin 2t$. Find the average value of f on the interval $[0, \pi]$.

$$\underbrace{\frac{1}{\pi - 0}}_{+1}\underbrace{\int_0^\pi t\sin 2t\,dt}_{+1} = \frac{1}{\pi}\int_0^\pi t\sin 2t\,dt = \frac{1}{\pi}\left[-t\tfrac{1}{2}\cos 2t + \int \tfrac{1}{2}\cos 2t\,dt\right]_0^\pi \quad +1$$

$$= \frac{1}{\pi}\left[-t\tfrac{1}{2}\cos 2t + \tfrac{1}{2}\cdot\tfrac{1}{2}\sin 2t\right]_0^\pi \quad +1$$

IP +1

+1

$$\left[\begin{array}{ll} u = t & v = -\tfrac{1}{2}\cos 2t \\ du = dt & dv = \sin 2t\,dt \end{array}\right.$$

notation +1

+2

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\cos 2\pi + \tfrac{1}{4}\sin 2\pi - (0 + \tfrac{1}{4}\sin 0)\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2} + 0 - 0 - 0\right]$$

$$= -\frac{1}{2} \quad +1$$

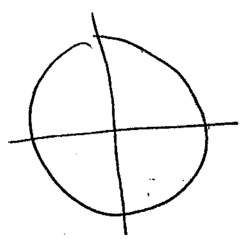
Check:

$$\frac{d}{dt}\left(-\tfrac{1}{2}t\cos 2t + \tfrac{1}{4}\sin 2t\right)$$

$$-\tfrac{1}{2}\left[-t\sin 2t\cdot 2 + \cos 2t\right] + \tfrac{1}{4}\cdot 2\cos 2t$$

$$t\sin 2t - \cancel{\tfrac{1}{2}\cos 2t} + \cancel{\tfrac{1}{2}\cos 2t}$$

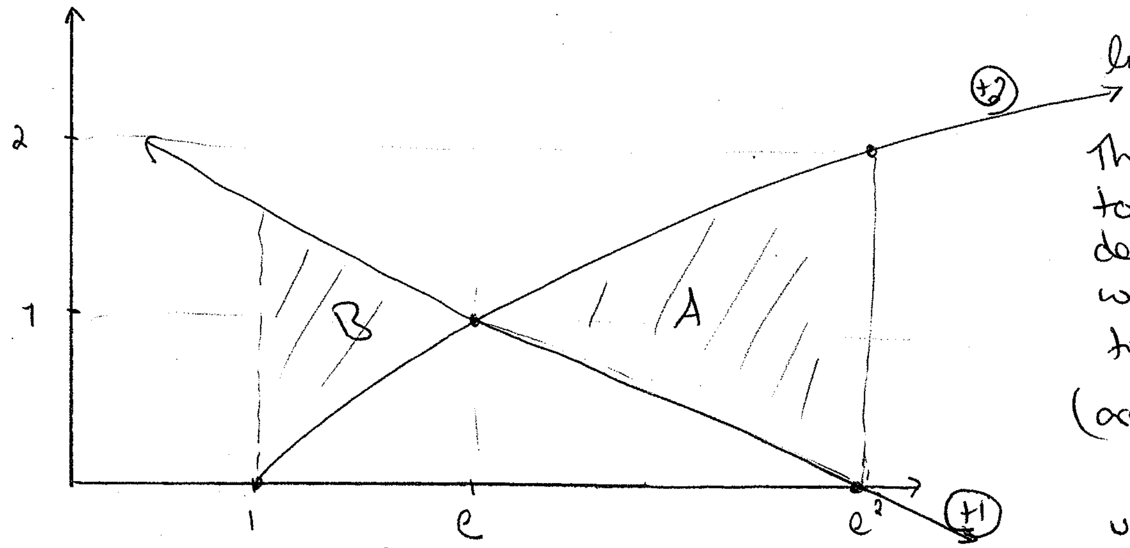
✓



(+1) $\frac{-\cancel{e}}{\cancel{e}(e-1)} + \frac{e}{e-1} = \frac{-1+e}{e-1} = 1$

$\frac{-e^2}{e(e-1)} + \frac{e}{e-1} = \frac{-e}{e-1} + \frac{e}{e-1} = 0$

4. (a) [8] Interpret $\int_1^{e^2} \ln x - (\frac{-1}{e(e-1)}x + \frac{e}{e-1})dx$ as the area of a region, by sketching a graph. *Hint: $x = e$ and $x = e^2$ are good points to plot.*



The area assoc. to the above def integral would correspond to area (+1)

(area A) - (area B) (+2)

understandable (+1)

(b) [4] Interpret $\int_1^{e^2} |\ln x - (\frac{-1}{e(e-1)}x + \frac{e}{e-1})|dx$ as the area of a region.

The abs value insists the area taken will be positive thus the above definite integral corresponds to

|area A| + |area B| ~~(+2)~~ 2 area mentioned +2 ~~understandable (+1)~~

(c) [5] Explain how to evaluate $\int_1^{e^2} |\ln x - (\frac{-1}{e(e-1)}x + \frac{e}{e-1})|dx$, but do *not* perform the evaluation.

~~split (+3)~~

~~where split (+2)~~

split +2

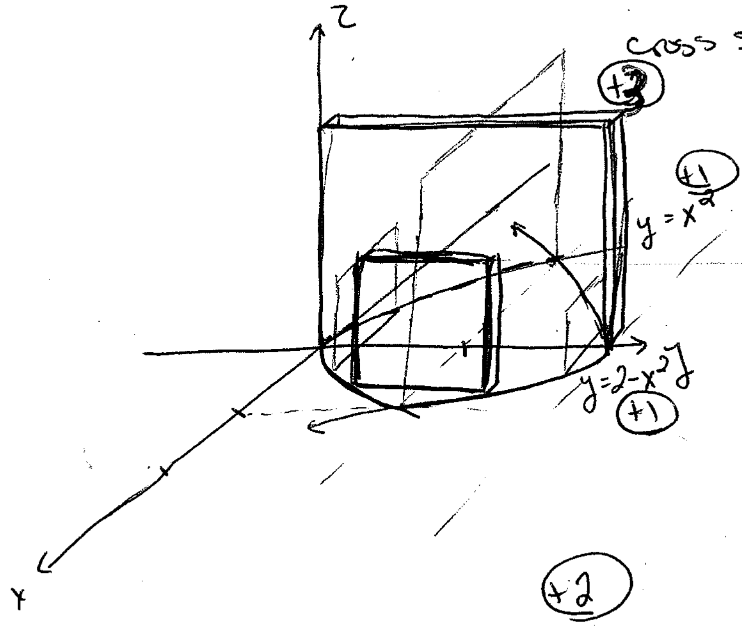
reorder +2

where split +1

$$\int_1^e \left(\frac{-1}{e(e-1)}x + \frac{e}{e-1}\right) - \ln x \, dx + \int_e^{e^2} \ln - \left(\frac{-1}{e(e-1)}x + \frac{e}{e-1}\right) dx$$

~~+1~~

5. [15] Consider the solid whose base is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$. The cross-sections perpendicular to the x-axis are squares with one side lying along the base. Sketch the volume and then find its volume.



notation +1

+2

$$\lim_{\Delta x \to 0} \sum (\text{area of sq}) \Delta x$$

$$\lim_{\Delta x \to 0} \sum (\text{top line} - \text{bottom line})^2 \Delta x$$

$$\lim_{\Delta x \to 0} \sum (2 - x^2 - x^2)^2 \Delta x$$

+1

$$\int_{-1}^{1} (\underbrace{2 - x^2 - x^2}_{\text{order}})^2 dx = 2\int_0^1 (2 - 2x^2)^2 dx = \cancel{2\left[2x - \frac{2}{3}x^3\right]_0^1}$$

order +1

bounds +1

$$= \cancel{2\left[2 - \frac{2}{3}\right] - 2\left[\frac{6-2}{4}\right]}$$

$$= 2\int_0^1 (4 - 8x^2 + 4x^4)\,dx$$ int +1

$$= 2\left[4x - \frac{8}{3}x^3 + \frac{4}{5}x^5\right]_0^1$$ eval +2

$$= 2\left[4 - 8/3 + 4/5\right] = 2\left[\frac{60 - 40 + 12}{15}\right]$$

$$= 2\left[\frac{32}{15}\right] = \frac{64}{15}.$$ +1

$$\begin{array}{r} 15 \\ \times 4 \\ \hline 60 \end{array}$$

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6. [10] Recall the Mean Value Theorem from first term calculus:

If g is a continuous function on the closed interval $[a, b]$, and differentiable on the open interval (a, b), then there is a number c between a and b such that

$$g'(c) = \frac{g(b) - g(a)}{b - a}$$

Prove that if f is continuous on $[a, b]$, then there exists a number c between a and $b]$ such that

$$\int_a^b f(x)dx = f(c)(b - a)$$

Hint: consider $F(t) = \int_a^t f(x)dx$.

Know to use.

Since $F(t)$ is con't on $[a,b]$ & diff on (a,b) [by FTC I], the mean value Thm for 251 ~~gives~~ says $\exists$ $c \in [a,b]$ $\ni$

(+2) $F'(c) = \dfrac{F(b)-F(a)}{b-a}$.

Consider the RHS:

(+2) $\dfrac{F(b)-F(a)}{b-a} = \dfrac{\int_a^b f(x)dx - \int_a^a f(x)dx}{b-a}$ (+2)

$= \dfrac{\int_a^b f(x)dx}{b-a}$ since $\int_a^a f(x)dx = 0$

The LHS

(+2) $F'(c) = \left(\frac{d}{dt}\int_a^t f(x)dx\right)_{t=c}$ (+2) by FTC I used ti mention ti

$= f(t)\Big|_{t=c}$

$= f(c)$.

conclude (+1)

Since LHS = RHS

$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$

$\Rightarrow (b-a)f(c) = \int_a^b f(x)dx$ //