MIDTERM 1 # Math 252 Fall 2007

NAME: KEY

1. [3] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let a and b be constants. f, g are cont

False was ok

T F $\int_a^b f(x) - g(x)dx = \int_a^b f(x)dx - \int_a^b g(x)dx$

T F $\int_a^b f(x)g(x)dx = \int_a^b f(x)dx \int_a^b g(x)dx$

T F $\int_{-1}^1 \frac{1}{x^2}dx = \frac{-1}{x}|_{-1}^1 = \frac{-1}{1} - \frac{-1}{-1} = -2$

not cont

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [4] *Carefully* write down the first Fundamental Theorem of Calculus.

If f is a cont. function (+1)

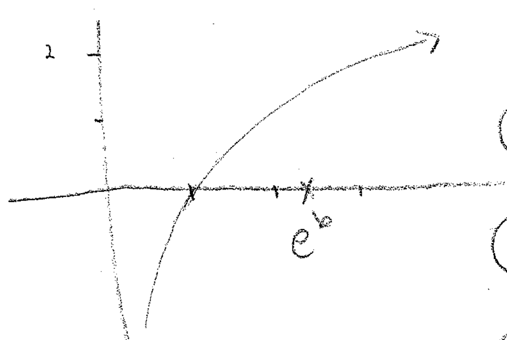
$\underbrace{\frac{d}{dx}}_{(+1)} \underbrace{\int_a^x}_{} \underbrace{f(t)dt}_{t\text{'s }(+1)} = f(x)$

important words: if, then

~~correct~~

language (+1)

3. [4] Find the equation of the line that is tangent to the graph of $y = \ln x$ at $x = e^b$ for some constant b.



(+1) pt on graph $(e^b, \ln e^b) = (e^b, b)$

(+1) $\frac{d}{dx} \ln x = \frac{d}{dx}\int_1^x \frac{1}{t}dt = \frac{1}{x}$

(+1) at $x = e^b$ slope is $\frac{1}{e^b}$

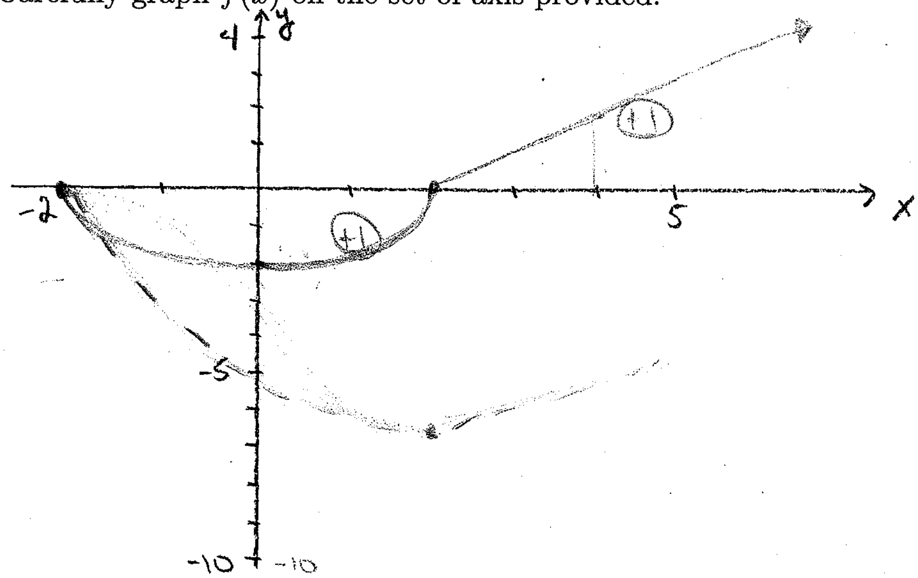
(+1) $y - b = \frac{1}{e^b}(x - e^b)$

$$f(x) = \begin{cases} -\sqrt{4-x^2}; \text{ if } -2 \le x \le 2 \\ x-2; \text{ if } 2 < x \end{cases}$$

$(-y)^2 + x^2 = 4$

4. Refer to the above definition of $f(x)$ to answer the following questions.

   (a) [2] Carefully graph $f(x)$ on the set of axis provided.



   (b) [3] Use your above graph to find $\int_{-2}^{4} f(x)dx$.

$$-\underbrace{\tfrac{1}{2}(\pi(2)^2)}_{(+1)} + \underbrace{\tfrac{1}{2}(2 \cdot 2)}_{(+1)} = \underbrace{-2\pi + 2}_{(+1)}$$

   (c) [4] Sketch the graph of $\int_{-2}^{x} f(t)dt$ above and clearly mark it as such.

(+1) Start @ 0 (+1) shape

(+1) neg dir (+1) becomes linear + up

5. [3] Find $\frac{d}{dx}\int_0^{x^2}\sqrt{1+r^3}\,dr$

(+1) $\left[\begin{array}{l} g(x) = x^2 \\ f(x) = \int_0^x \sqrt{1+r^3}\,dr \end{array}\right.$ $\quad (f(g(x)))' = f'(g(x))g'(x)$

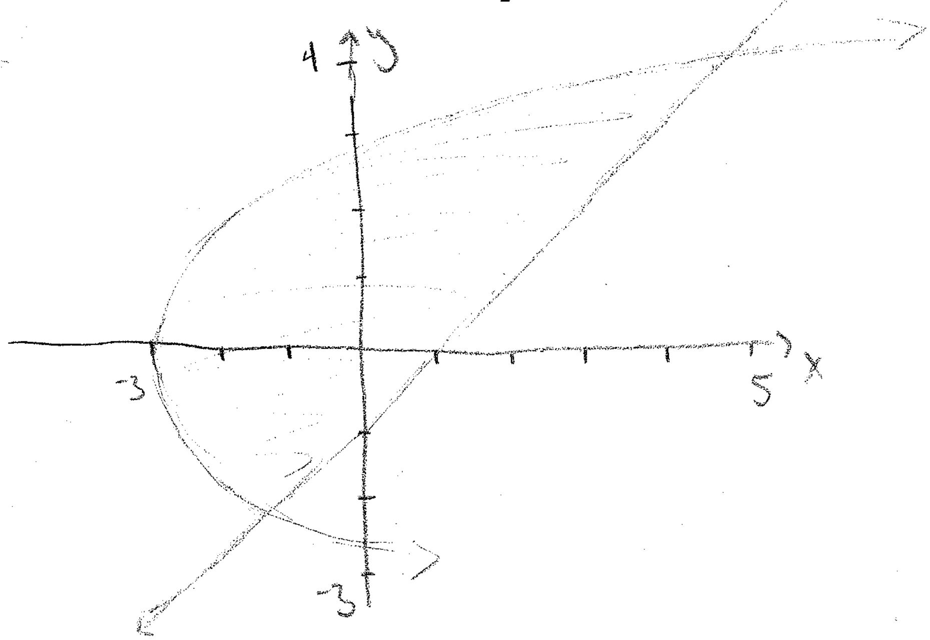
$\underline{\sqrt{1+(x^2)^3}}\ 2x = 2x\sqrt{1+x^6}$

(+1) (+1)

6. [5] Set up the definite integral ~~what~~ that gives the area of the region bounded by $y+1=x$ and the parabola $\frac{1}{2}y^2-3=x$. Do *not* evaluate the integral.

$y = x-1$

$y^2 = (x+3)2$
$= 2x+6$



graph (+2)

$\frac{1}{2}y^2 - 3 = y+1$

$\frac{1}{2}y^2 - y - 4 = 0$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$4, -2$

looking for intersection pts (+1)

$$\int_{-2}^{4} (y+1) - \left(\tfrac{1}{2}y^2 - 3\right) dy$$

correct one on top (+1)

bounds (+1) correct

7. [6 each] Evaluate *ONLY TWO* of the following. Indicate clearly which two you want graded by completely striking the problem you do not want graded.

(a) $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \sin x}{1+x^6}\, dx$

odd function $\therefore$ 0

+3 +3

(b) $\displaystyle\int x^3\sqrt{x^2+1}\, dx$

$\int x^2 \sqrt{u}\; x\,dx = \frac{1}{2}\int x^2 \sqrt{u}\, du$ (+1)

(+1) $u = x^2+1 \Rightarrow u-1 = x^2$

$du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int (u-1)\sqrt{u}\, du$ (+1)

$= \frac{1}{2}\int u^{3/2} - u^{1/2}\, du$ (+1)

$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$ (+1)

$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + c$

(+1) sub in x's

(c) $\displaystyle\int \frac{e^x}{1+e^{2x}}\, dx$

(+2) $u = e^x$

$du = e^x\, dx$

$\int \frac{e^x dx}{1-(e^x)^2} = \int \frac{1}{1-u^2}\, du$ (+1)

$= \tan^{-1} u + C$ (+1) (+1)

$= \tan^{-1}(e^x) + c$

(+1) sub in x's

$\frac{d}{dx}(e^{2x}) \neq 2e^x$

$2e^{2x}$

8. [10] Kobayashi has won the hot dog-eating world championship six times. Recently he challenged a giant bear to a 3 minute hot dog-eating contest. Kobayashi found that the rate he can eat hot dogs goes down as time goes by and can be modeled by $k(t) = \frac{12}{(t+1)^2} + 24$, were t is measured in minutes. The bear isn't quite as used to the system and seems to start with a slower rate that gets larger and is well modeled by $b(t) = 8\cancel{(}t^3 + 20\cancel{)}$. Find out how many hot dogs Kobayashi and the bear eat and determine who won the context.

# of hot dogs eaten correspond to $\int_0^3$

Kobayashi

$\int_0^3 \frac{12}{(t+1)^2} + 24 \, dt$ +1

$12\int_0^3 \frac{1}{(t+1)^2} \, dt + 24t\Big|_0^3$

$u = t+1$

$du = dt$

$12\int_1^4 \frac{1}{u^2} \, du + 72$

1 point for sub

$12 \; \frac{-1}{u}\Big|_1^4 + 72$

+1

$12\left(-\frac{1}{4} + \frac{+1}{1}\right) + 72$

$-3 + 12 + 72$

$9 + 72$

81 +1

4

Bear

$\int_0^3 8(t^3+20) \, dt$ +1

should have had $8t^3 + 20$

$8\int_0^3 t^3 \, dt + \cancel{8} \cdot 20t\Big|_0^3$

$8 \; \frac{1}{4}t^4\Big|_0^3 + \cancel{160 \cdot 3} \; 20 \cdot 3$

+1

$2 \cdot 3^4 + \cancel{160 \cdot 3} \;\; 60$

$2 \cdot 81 + \cancel{480} \;\; 60$

$162 + \cancel{480} \;\; 60$

$\cancel{642} \;\; 222$

+1

4

The Bear won! +1