3:00
3:10

Quiz 7
Math 251

Name: KEY

1. [2] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f'' be a continuous function defined on $(-\infty, \infty)$.

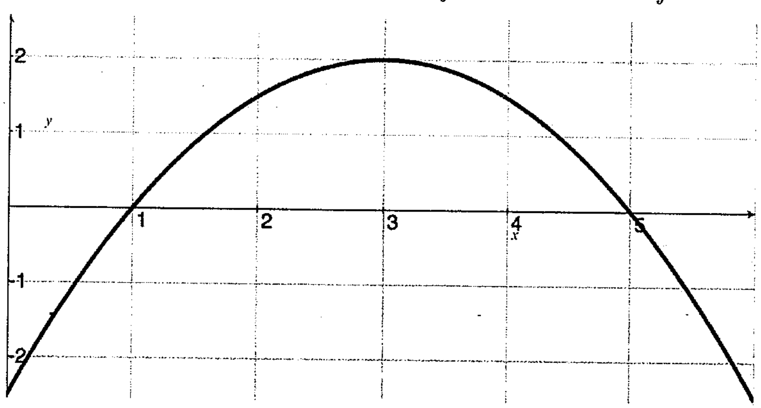
   T F If $f'(2) = 0$ and $f''(2) = -5$, then f has a local maximum when $x = 2$.

   T F If $f'(6) = 0$ and $f''(6) = 0$, then f will not have a local extrema at $x = 6$.

$f(x) = (x-6)^4$
min at $x = 6$
but
$f'(x) = 4(x-6)^3$
$f''(x) = 12(x-6)^2$
$f'(6) = f''(6) = 0$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [2] The graph of the *derivative* f' of a function f is shown.



(a) On what interval/s is f increasing?

$(1, 5)$

(b) At what value/s of x does f have a local minimum?

at $x = 1$

$\searrow \rightarrow \nearrow$

$\nearrow \rightarrow \searrow$

3. [6] Consider the function $f(x) = \frac{x^2}{x^2-1}$. $= \frac{x^2}{(x+1)(x-1)}$ Domain $x \neq 1, -1$

[1] (a) *Justify* the existence or nonexistence of any vertical asymptotes.

$\lim_{x\to 1^-} f(x) = -\infty$ $\lim_{x\to -1^-} f(x) = +\infty$

$\lim_{x\to 1^+} f(x) = +\infty$ $\lim_{x\to -1^+} f(x) = -\infty$

[1] notation

[1] (b) *Justify* the existence or nonexistence of any horizontal asymptotes.

$\lim_{x\to\infty} \frac{x^2}{x^2-1}\frac{(1/x^2)}{(1/x^2)} = \lim_{x\to\infty} \frac{1}{1-1/x^2} = 1$ $\lim_{x\to-\infty} \frac{x^2}{x^2-1} = \lim_{x\to-\infty} \frac{1}{1-1/x^2} = 1$

H.A at $y=1$

[2] (c) Find the intervals when f is increasing.

$f'(x) = \frac{(x^2-1)2x - x^2 2x}{(x^2-1)^2} = \frac{\cancel{2x^3} - 2x - \cancel{2x^3}}{(x^2-1)^2} = \frac{-2x}{(x+1)^2(x-1)^2}$ finding intervals to check (+1)

$+ \quad + \quad - \quad -$

$-1 \quad 0 \quad 1$

roots at $x=0$
not def at $x=-1$, or 1 got it (+1)

inc $(-\infty,-1)$ and $(-1,0)$

~~(d)~~ Find the intervals when f is concave up.

thrown out in the interest of time.

$f''(x) = \frac{-(x+1)^2(x-1)^2\cdot 2 + 2x\cdot 2(x^2-1)\cdot 2x}{[(x^2-1)^2]^2} = \frac{-2(x^2-1)^{\cancel{2}} + 8x^2\cancel{(x^2-1)}}{(x^2-1)^{\cancel{4}}}$

$= \frac{-2(x^2-1)+8x^2}{(x^2-1)^3} = \frac{-2x^2+2+8x^2}{(x^2-1)^3}$ these are no roots
not def at $x=-1$ or 1

$= \frac{6x^2+2}{(x^2-1)^3}$ $+ \quad - \quad +$ CU $(-\infty,-1)$ and $(1,\infty)$

[1] (e) Using the above information you found above, sketch the graph of f. (You need not run through all the steps covered in §4.5! You only need to sketch a graph that is consistent with the information you found above.)