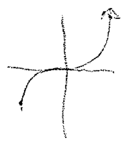
Quiz 6
Math 251

Name: KEY

1. [2] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function.

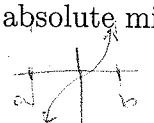
$x^3 = y$



T **F** If $f'(c) = 0$, then f has a local maximum or minimum at c.

could have vert. asy.

T **F** If f is continuous on (a, b), then f attains an absolute maximum value $f(c)$ and an absolute minimum calue $f(d)$ at some numbers c and d in (a, b).



Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [4] Find the following:

$\lim_{x\to 0} \frac{x + \sin x}{x + \cos x}$ $\frac{0+0}{0+1} = \frac{0}{1}$

by limit law #5

$= \frac{\lim_{x\to 0}(x+\sin x)}{\lim_{x\to 0}(x+\cos x)}$

$= \frac{0+0}{0+1} = \frac{0}{1} = 0$ (1)

notation (+1)

$\lim_{t\to 0} \frac{5^t - 3^t}{t}$ "$\frac{0}{0}$"

(1) 2+1 $= \lim_{t\to 0} \frac{5^t \ln 5 - 3^t \ln 3}{1}$

$= \ln 5 - \ln 3$ (1)

$\frac{d}{dx}(a^x) = ?$
$= a^x \ln a$

$y = a^x$
$\ln y = x \ln a$
$\frac{1}{y}\frac{dy}{dx} = \ln a$
$\frac{dy}{dx} = y \ln a = a^x \ln a$

3. [4] Consider the function $g(x) = 1 + 2x + x^3 + 4x^5$

(a) Show that g has at least one root. Explain your reasoning clearly and *cite* ~~and~~ any theorems you use.

Note: $g(1) = 1+2+1+4 = 8$ and $g(-1) = 1-2-1-4 = -6$

Recall the IVT: +1

Since g is a cont function on the interval $[-1, 1]$ and 0 is between $g(1)$ & $g(-1)$, the ~~[illegible]~~ IVT says there exists a c between -1 & 1 so that $g(c) = 0$.

used +1 correctly

Thus the IVT $\Rightarrow$ c is a root of g.

(b) Show that g has at most one root. Explain your reasoning clearly and *cite* ~~and~~ any theorems you use.

Assume towards contradiction that g has at least two roots. Label one α & the other β.

Thus $g(\alpha) = 0 = g(\beta)$.

Recall Rolle's thm:

~~Sing~~ +1

Since g is cont on all $\mathbb{R}$, g is cont on $[\alpha, \beta]$

Since g is diff. on all $\mathbb{R}$, g is diff on (α, β).

By set up $g(\alpha) = g(\beta)$.

Rolle's thm. then says there exists a γ between α & β so that

$$g'(\gamma) = 0.$$

But $g'(x) = 2 + 3x^2 + 20x^4 > 0$ for all x. ~~to~~

Our assumption of the existence of 2 roots must have therefore been false.