Quiz 4
Math 251

Name: KEY

Show *all* your work (algebraically or geometrically) for each and ~~simplify~~. No credit is given without supporting work.

1. [5] Use any results covered in class to find the following:

notation (+1)

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

(+1)

$$\lim_{x \to 0} \frac{\sin 3x}{x} = \lim_{x \to 0} \frac{3}{3} \frac{\sin 3x}{x}$$

$$= \lim_{x \to 0} 3 \frac{\sin 3x}{3x}$$

$$= 3 \lim_{x \to 0} \frac{\sin 3x}{3x}$$

$$= 3$$

(+1)

$$\frac{d}{dx}\left(\frac{\sin x}{x^2}\right)$$

$$= \frac{x^2 \, d/dx(\sin x) - \sin x \, d/dx(x^2)}{(x^2)^2}$$

$$= \frac{x^2 \cos x - \sin x \cdot 2x}{x^4}$$

$$= \frac{x^2 \cos x - 2x \sin x}{x^4}$$

(+1)

or

$$\frac{x \cos x - 2 \sin x}{x^3}$$

$$\frac{d}{dx}\left(2^{\sin \pi x}\right)$$

$f(x) = 2^x \qquad f'(x) = 2^x \ln 2$

$g(x) = \sin \pi x \qquad g'(x) = *$

$f(g(x)) = 2^{\sin \pi x}$ ✓

$$d/dx\left(2^{\sin \pi x}\right) = f'(g(x))g'(x)$$

$$= f'(\sin \pi x) \cdot \pi \cos \pi x$$

(+1)

$$\boxed{= 2^{\sin \pi x} \cdot \ln 2 \cdot \pi \cos \pi x}$$

* $d/dx(\sin \pi x)$

$f(x) = \sin x \qquad f'(x) = \cos x$

$g(x) = \pi x \qquad g'(x) = \pi$

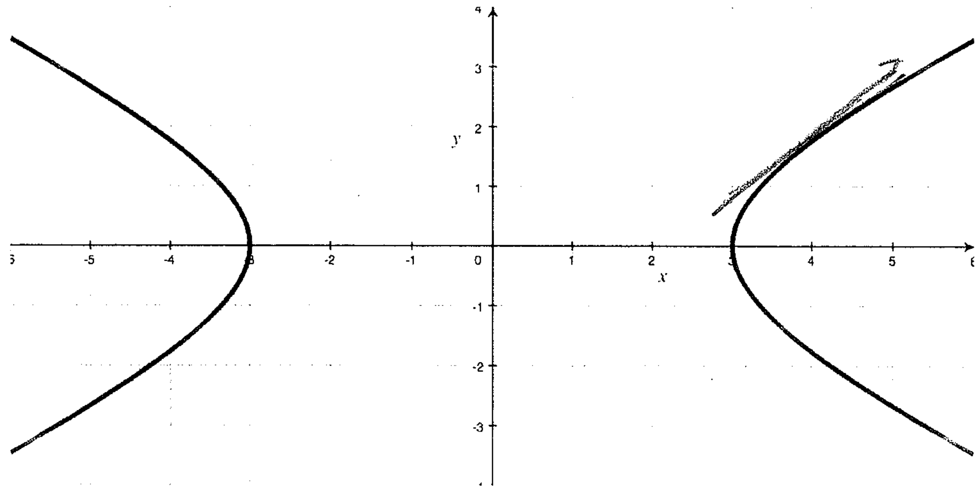
$f(g(x)) = \sin \pi x$ ✓

$$d/dx(\sin \pi x) = f'(g(x))\,g'(x)$$

$$= f'(\pi x) \cdot \pi$$

$$= (\cos \pi x)\,\pi$$

2. [5] . The graph of $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is an example of a hyperbola and is shown below. Find the equation of the line that is tangent to the graph below at $(4, \frac{2\sqrt{7}}{3})$.



need $\left.\frac{dy}{dx}\right|_{(4,\frac{2\sqrt{7}}{3})}$

d/dx of $\frac{x^2}{9}$ and 1 (+1)
d/dx of y^2 (+1)

$\frac{d}{dx}\left(\frac{x^2}{9} - \frac{y^2}{4}\right) = \frac{d}{dx}(1)$

$\frac{2}{9}x - \frac{1}{4}\frac{d}{dx}(y^2) = 0 \quad \rightsquigarrow \quad \frac{2}{9}x - \frac{1}{4}2y\frac{dy}{dx} = 0$

$f(x) = x^2 \quad f'(x) = 2x$
$g(x) = y \quad g'(x) = dy/dx$
$f(g(x)) = f(y) = y^2$ ✓
$d/dx(y^2) = f'(g(x))g'(x)$
$= f'(y) \cdot \frac{dy}{dx}$
$= 2y\frac{dy}{dx}$

so $\left.\frac{dy}{dx}\right|_{(4,\frac{2\sqrt{7}}{3})}$

$\frac{2 \cdot 4}{9} - \frac{1}{\cancel{4}}\left(\frac{\cancel{2}\sqrt{7}}{3}\right)\left.\frac{dy}{dx}\right|_{(4,\frac{2\sqrt{7}}{3})} = 0$

plug in pt $(4, \frac{2\sqrt{7}}{3})$ to the derivative (+1)

$\frac{8}{9} = \frac{\sqrt{7}}{3}\left.\frac{dy}{dx}\right|_{(4,\frac{2\sqrt{7}}{3})}$

$\Rightarrow \frac{dy}{dx} = \frac{24}{9\sqrt{7}} = m$

so $y - \frac{2\sqrt{7}}{3} = \frac{24}{9\sqrt{7}}(x-4)$

looking for line (+1)
plug in pt $(4, \frac{2\sqrt{7}}{3})$ (+1) for line.