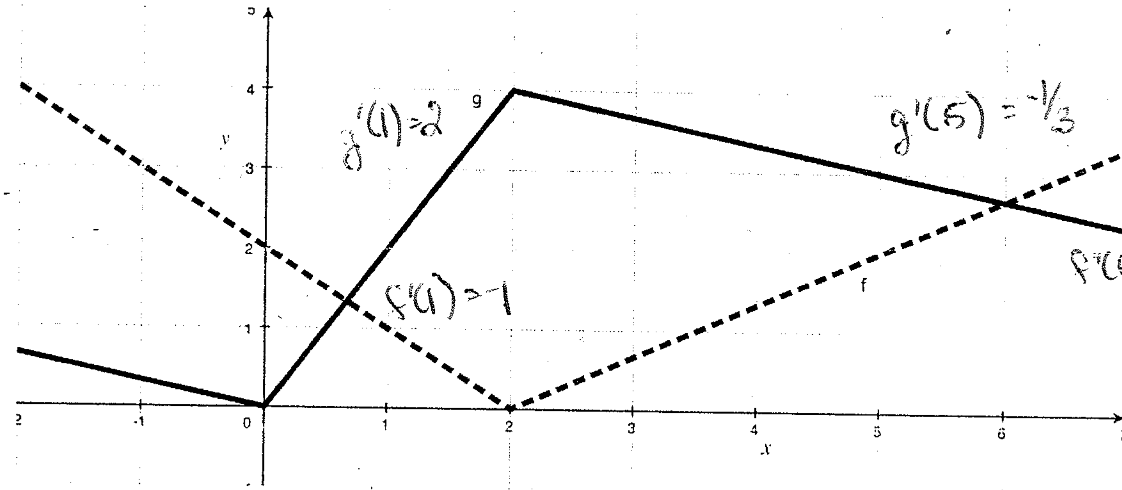
Quiz 3
Math 251

Name: KEY

Show *all* your work (algebraically or geometrically) for each and simplify. No credit is given without supporting work.

1. [2] Let $u(x) = 2f(x) - g(x)$ and $v(x) = f(x)/g(x)$ where the graphs of f and g are given below.



Find the following:

$u'(1) = 2f'(1) - g'(1) = 2\cdot(-1) - 2$
$= -2-2 = -4$ (+1)

$v'(5) = \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)\Big|_{x=5} = \frac{g(5)f'(5) - f(5)g'(5)}{(g(5))^2} = \frac{3\cdot\frac{2}{3} - 2\cdot\frac{-1}{3}}{3^2} = \frac{2+\frac{2}{3}}{9} = \frac{8}{3}\cdot\frac{1}{9} = 8/27$ (+1)

2. [3] Let $f(x) = 2xe^x$. Find the equation of the tangent line to the graph of f when $x = 0$.

looking for m & b in $y = mx+b$.

Note $m = f'(0)$

(+1) and $f'(x) = \frac{d}{dx}(2xe^x) = 2\frac{d}{dx}(xe^x) = 2\left(x\frac{d}{dx}e^x + \left(\frac{d}{dx}x\right)e^x\right)$
$= 2(xe^x + e^x)$

(+1) so $m = f'(0) = 2(0\cdot e^0 + e^0) = 2\cdot 1$

So we have $y = 2x+b$.

(+1) Note that the point $(0, f(0))$ is both on the graph of f an our tang line. Since $f(0) = 2\cdot 0e^0 = 0$ we have $0 = 2\cdot 0 + b \Rightarrow b = 0$.

Thus our equation is $y = 2x$.

3. [5] *Prove* the following using only the definition of a derivative and limit properties. Clearly explain each of your steps.

If c is a real number and f is a differentiable function for all x, then

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$$

$$\frac{d}{dx}(cf(x)) = \lim_{h\to 0} \frac{cf(x+h) - cf(x)}{h} \quad \text{by def. of } \frac{d}{dx}$$

$$= \lim_{h\to 0} \frac{c(f(x+h) - f(x))}{h} \quad \text{by alg.}$$

+1

$$= c\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \quad \text{by prop. of limits}$$

$$= c\frac{d}{dx}f(x) \quad \text{by def of } \frac{d}{d}.$$

Thus $\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$.

notation +1
used def of $\frac{d}{dx}$ +1
sense/logic +1
string from one end to other +1