FINAL

Math 251

Practice

Note: This is a practice final and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. [] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be differentiable functions and h be a constant.

   T F $\frac{x+h}{2x} = \frac{1+h}{x}$

   T F $\sqrt{x^2+h^2} = x+h$

   T F $\lim_{x\to r} f(x) = f(r)$ for all r in the domain of f.

   T F If $\lim_{x\to r} g(x) = 0$, then $\lim_{x\to r} \frac{f(x)}{g(x)}$ does not exist.

   T F $\frac{d}{dx}(\frac{1}{x}) = -1$

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. Using the *definition*, find the derivative of $f(x) = \sqrt{4x - \frac{3}{2}}$

3. Given that $f(x)$ is a differentiable function and that a and k are constants, complete the following Derivative Rules:

(a) $\frac{d}{dx}(a^k) =$

(b) $\frac{d}{dx}(f(x)^k) =$

(c) $\frac{d}{dx}(a^{f(x)}) =$

(d) $\frac{d}{dx}\ln(f(x)) =$

(e) $\frac{d}{dx}(e^{f(x)}) =$

(f) $\frac{d}{dx}(\log_a(f(x)) =$

4. Prove if f and g are differentiable, then $\frac{d}{dx}(f-g) = \frac{d}{dx}f - \frac{d}{dx}g$. *Hint:* use the *definition* of a derivative.

5. Let $f(x) = \begin{cases} \sqrt{1-(x+3)^2} & \text{if } -4 \leq x \leq -2 \\ 1 & \text{if } -2 < x < 2 \\ -(x-3)^2+2 & \text{if } 2 < x < 4 \\ -3 & \text{if } 4 < x \end{cases}$

Graph $f(x)$ and then graph $f'(x)$ below on its own set of axes. Afterwards, answer the following questions.

$\lim\limits_{x \to 4} f(x)$

$\lim\limits_{x \to -2^+} f(x)$

$\lim\limits_{x \to -2^-} f'(x)$

$\lim\limits_{x \to \infty} f(x)$

6. What is $\lim\limits_{x \to 0}(\frac{-4}{x})$? Explain your answer.

7. Find $\lim_{x \to 0} x^4 sin(\frac{1}{x})$. Recall that $sin(\frac{1}{x})$ oscillates between -1 and 1 as it gets closer to zero. Explain your reasoning.

8. Prove that the function $f(x) = x^3 - 5$ has a *fixed point*. (i.e. show that there exists a point p such that $f(p) = p$)

9. Find $\frac{dy}{dx}$ for each of the following:

$y = x \sin \frac{1}{x}$

$y = x^{x^x}$

$x^2y^2 = 4 - y \arctan(5x)$

$y = \sqrt{x}e^{x^7}(x^6 + 3)^{10}$

10. Suppose that height of a ball from the floor at time t, is described by the equation $H(t) = -t^2 + 7t + 8$.

(a) When is the ball on the floor?

(b) Find when the ball is at its maximum height.

(c) Find the acceleration of the ball at $t = 1$.

(d) Find the total distance traveled from $t = 0$ to $t = 5$.

11. Find the following *limits*

$$\lim_{x\to\infty} \frac{\sqrt{x^2+2}}{\sqrt{2x^2+1}} \qquad\qquad \lim_{x\to\infty} x^x$$

12. Fill out the following and then graph $f(x) = 2\cos x + \sin 2x$

- Domain:
- x-intercepts:
- y-intercepts:
- Symmetry of $f(x)$:
- vertical asymptotes:
- horizontal asymptotes:
- extrema (both x and y coordinates)
- intervals $f(x)$ is increasing:
- pts. of inflection (both x and y coordinates)
- intervals $f(x)$ is concave up
- Graph $f(x)$

13. A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of $2\text{m}^3/\text{min}$, find the rate at which the water level is rising when the water is 3m deep.

    Recall the volume of a cone is $\frac{1}{3}\pi r^2 h$ where r is the radius of the base and h is the height of the cone.

14. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.