MIDTERM 1 # Math 251 Practice

Note: This is a practice midterm and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

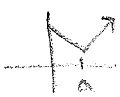
1. [] TRUE/FALSE: Let f and g be functions. Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T F $\frac{3x+y}{3z} = \frac{x+y}{z}$

T F $(x+y)^2 = x^2 + y^2$ $\quad (x+y)^2 = (x+y)(x+y) = x^2 + xy + xy + y^2$

T F $\displaystyle\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} f(x)}$ for all a $\quad$ typo: was intended to be $\dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ which would still be false

T F If $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} g(x) = \infty$, then $\lim_{x\to\infty}[f(x) - g(x)] = 0$. $\quad$ consider $f(x) = x^2$, $g(x) = x^4$



T F If f is continuous at a, then f is differentiable at a.

Thm T F If f is differentiable at a, then f is continuous at a.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. Find a formula for a function that has vertical asymptotes $x = 2$ and $x = 0$ and horizontal asymptote at $y = -1$.

$$\frac{-x^2-1}{x^2-2x} = \frac{-(x^2+1)}{(x-2)x}$$

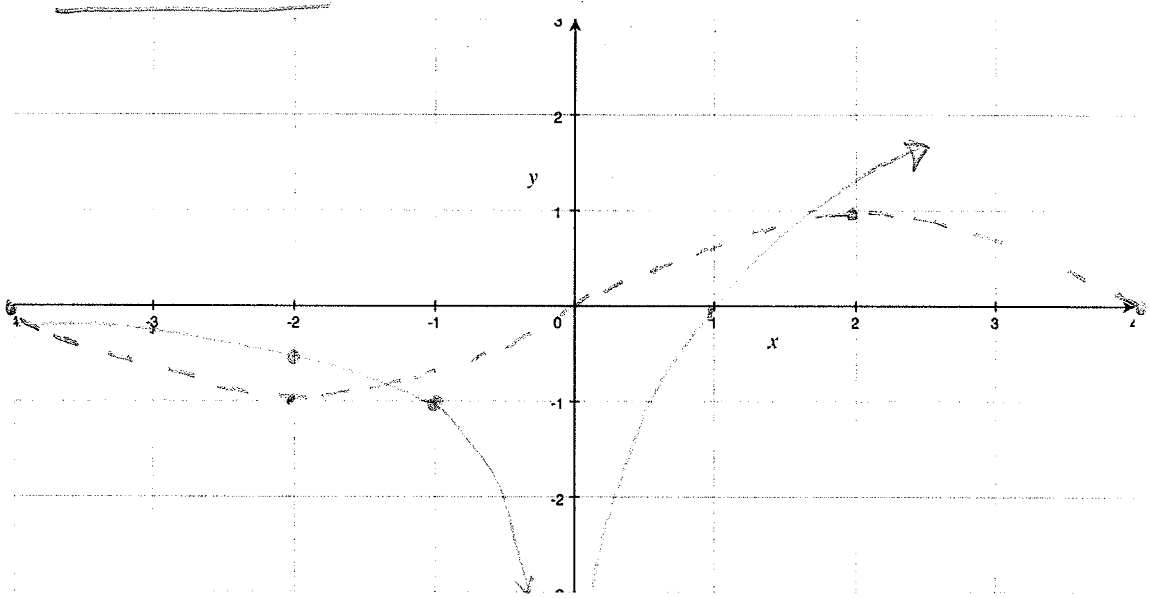
note: $\displaystyle\lim_{x\to\infty} \frac{-(x^2+1)}{x(x-2)} = \lim_{x\to\infty} \frac{-1-\frac{1}{x^2}}{1-\frac{2}{x}} = -1$

$$\left.\begin{array}{l} \displaystyle\lim_{x\to 2^-} \frac{-x^2-1}{x(x-2)} = \infty \\ \displaystyle\lim_{x\to 2^+} \frac{-x^2-1}{x(x-2)} = -\infty \end{array}\right\} \Rightarrow \text{v.a. at } x=2$$

$$\left.\begin{array}{l} \displaystyle\lim_{x\to 0^-} \frac{-x^2-1}{x(x-2)} = -\infty \\ \displaystyle\lim_{x\to 0^+} \frac{-x^2-1}{x(x-2)} = +\infty \end{array}\right\} \Rightarrow \text{v.a. at } x=0$$

3. Given the rules of f and g below, graph both functions on the axis provided and evaluate the following

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0, \\ 4 & \text{if } x = 0, \\ \ln x & \text{if } x > 0, \end{cases} \qquad g(x) = \sin(\tfrac{\pi}{4}x)$$



$\lim\limits_{x\to\infty} g(x)$

DNE

$\lim\limits_{x\to 0} f(x)$

$-\infty$

f(0) 4

$\lim\limits_{x\to 1}[\pi f(x) \times g(x)]$

$= \lim\limits_{x\to 1} \pi f(x) \cdot \lim\limits_{x\to 1} g(x) = \pi \cdot 0 \cdot \frac{1}{\sqrt{2}}$

$= 0$

$\lim\limits_{x\to -2}(2f(x) + g(x)) = 2\lim\limits_{x\to -2} f(x) + \lim\limits_{x\to -2} g(x)$

$= 2 \cdot \frac{-1}{2} + -1 = -1 - 1 = -2$

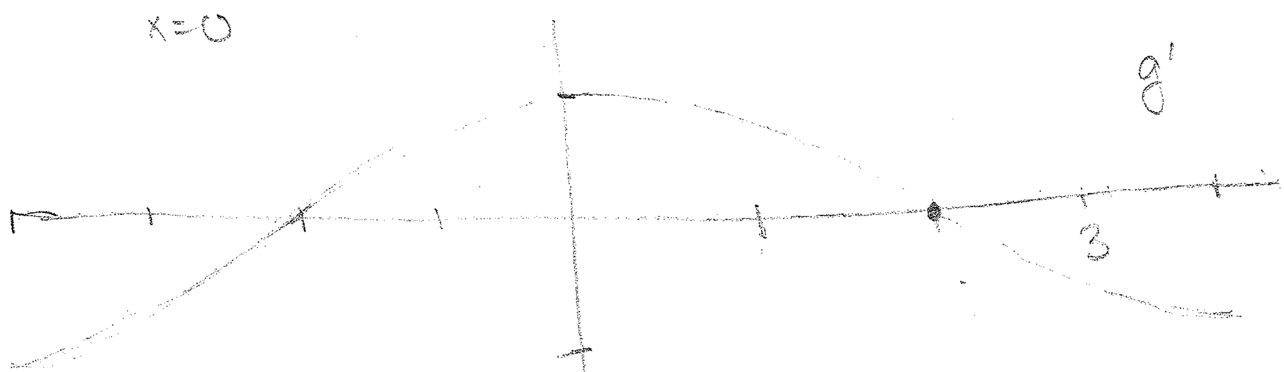
$g'(2)$

Considering the graph above @ the tang. line is horiz ∴ 0

List any values that f is *not* continuous at:

x = 0

Graph $g'(x)$



4. [] Find the limit if it exists, or explain why it does not exist.

$$\lim_{x\to 3}\frac{x^2-4x+3}{x^2-2x-3} = \lim_{x\to 3}\frac{(x-3)(x-1)}{(x-3)(x+1)}$$

$$= \lim_{x\to 3}\frac{x-1}{x+1}$$

$$= \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

$$\lim_{x\to\infty} e^{-2x}\sin x$$

note $-1 \le \sin x \le 1$

since $e^{-2x} > 0$

$$-e^{-2x} \le e^{-2x}\sin x \le e^{-2x}$$

since $\lim_{x\to\infty} e^{-2x} = 0$

and $\lim_{x\to\infty} -e^{-2x} = 0$

$\therefore$ by the squeeze thm

$$\lim_{x\to\infty} e^{-2x}\sin x = 0$$

$$\lim_{h\to 0}\frac{(1+h)^{-1}-1}{h}$$

$$= \lim_{h\to 0}\frac{\frac{1}{1+h}-1}{h} = \lim_{h\to 0}\frac{\left(\frac{1-1-h}{1+h}\right)}{h}$$

$$= \lim_{h\to 0}\frac{-h}{h(1+h)} = \lim_{h\to 0}\frac{-1}{1+h}$$

$$= -1$$

$$\lim_{x\to\infty}\arctan(x^2-x^4)$$

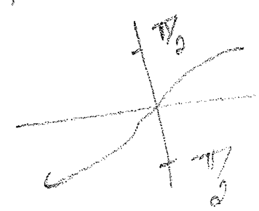
note $\lim_{x\to\infty}(x^2-x^4)$

$$= \lim_{x\to\infty} x^4\left(\frac{1}{x^2}-1\right) = -\infty$$

so $\lim_{x\to\infty}\arctan(x^2-x^4)$

$= \arctan \lim_{x\to\infty}(x^2-x^4)$ by cont.

$= -\pi/2$



$$\lim_{x\to 1}\frac{2x-2}{|x-1|}$$

recall $|x-1| = \begin{cases} x-1 & \text{if } x \ge 1 \\ -x+1 & \text{if } x < 1 \end{cases}$

$$\Rightarrow \lim_{x\to 1^-}\frac{2x-2}{|x-1|} = \lim_{x\to 1^-}\frac{2x-2}{x-1} = 2$$

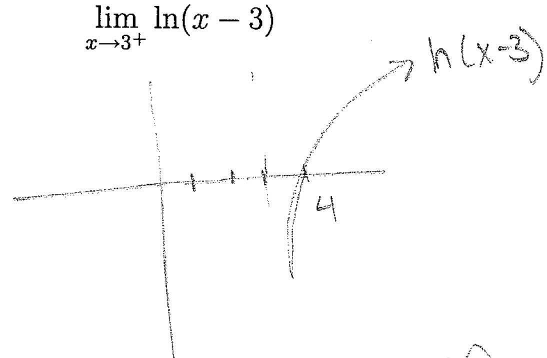
and

$$\lim_{x\to 1^+}\frac{2x-2}{|x-1|} = \lim_{x\to 1^+}\frac{2x-2}{-(x-1)} = -2$$

$2 \ne -2 \quad \therefore$

$$\lim_{x\to 1}\frac{2x-2}{|x-1|} \quad \text{DNE}$$

$$\lim_{x\to 3^+}\ln(x-3)$$



by the graph ... $-\infty$

5. Does $f(x) = 2x^3 + 6x^2 - 10x - 30$ have a root between 2 and 3? Explain your reasoning and cite theorems if you use any.

Note $f(2) = 2 \cdot 2^3 + 6 \cdot 2^2 - 10 \cdot 2 - 30$

$= 16 + 24 - 20 - 30 = 20 - 30 = -10 < 0$

and $f(3) = 2 \cdot 3^3 + 6 \cdot 3^2 - 10 \cdot 3 - 30$

$= 54 + 54 - 30 - 30 = 24 + 24 = 48 > 0$

So the IVT implies there exists a # between 2 & 3 that is a root.

6. [] Find the equation for the line tangent to the graph of $y = \frac{1}{(x-2)^2}$, when $x = 3$.

$f(x)$

$y = mx + b$.

finding m: $m = f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$

$= \lim_{h \to 0} \frac{\frac{1}{(3+h-2)^2} - \frac{1}{(3-2)^2}}{h}$

$= \lim_{h \to 0} \frac{\frac{1}{(1+h)^2} - \frac{1}{1}}{h}$

$= \lim_{h \to 0} \frac{1 - (1+h)^2}{h(1+h)^2}$

$= \lim_{h \to 0} \frac{\cancel{1} - \cancel{1} - 2h - h^2}{h(1+h)^2}$

$= \lim_{h \to 0} \frac{-2 - h}{(1+h)^2}$

$= \frac{-2}{1} \Rightarrow m = -2$

$y = -2x + b$ b/c $\left(3, \frac{1}{(3-2)^2}\right) = (3, 1)$ is on the line..

$1 = -2 \cdot 3 + b \Rightarrow 1 + 6 = b$

$7 = b$

$\therefore y = -2x + 7$

7. Suppose that the motion of a ball can be described by the equation $f(t) = t^2 + t - 3$. Find the instantaneous velocity of the ball after 4 seconds.

inst. velocity @ $t=4$ $= f'(4) = \lim_{h \to 0} \frac{f(4+h) - f(4)}{h}$

$= \lim_{h\to 0} \frac{((4+h)^2 + 4+h-3) - (4^2+4-3)}{h} = \lim_{h \to 0} \frac{16+8h+h^2+h+1-17}{h}$

$= \lim_{h\to 0} \frac{9h+h^2}{h} = \lim_{h\to 0} 9+h = 9$

so 9 units/sec.

8. [] Using the **definition**, find the derivative of $f(x) = \sqrt{2x - \frac{1}{2}}$

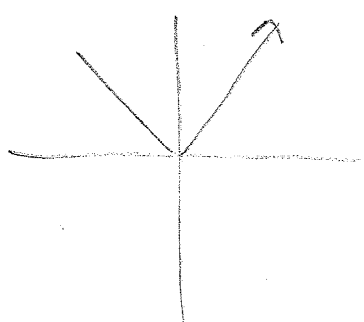
$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{2(x+h)-\frac{1}{2}} - \sqrt{2x-\frac{1}{2}}}{h}$

$= \lim_{h\to 0} \frac{2(x+h) - \frac{1}{2} - (2x - \frac{1}{2})}{h(\sqrt{2(x+h)-\frac{1}{2}} + \sqrt{2x-\frac{1}{2}})} = \lim_{h\to 0} \frac{2h}{h(\sqrt{2x+2h-\frac{1}{2}} + \sqrt{2x-\frac{1}{2}})}$

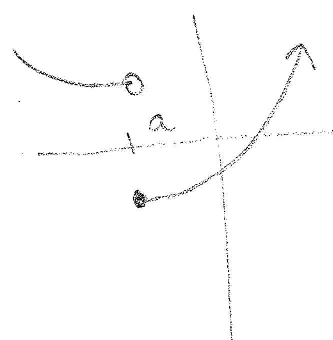
$\cancel{2x}+2h\cancel{-\frac{1}{2}}\cancel{-2x}\cancel{+\frac{1}{2}}$

$= \lim_{h\to 0} \frac{2}{\sqrt{2x+2h-\frac{1}{2}} + \sqrt{2x-\frac{1}{2}}} = \frac{2}{2\sqrt{2x-\frac{1}{2}}} = \frac{1}{\sqrt{2x-\frac{1}{2}}}$

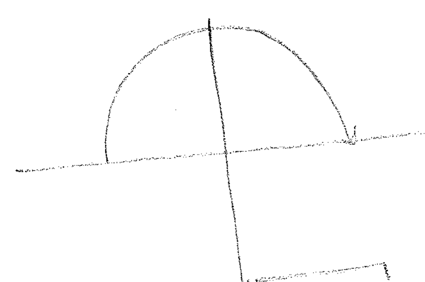
9. Describe 3 situations in which a function $f(x)$ could ***fail*** to be differentiable at a point.



$f(x) = |x|$ at $x=0$



if f jumps at a point.



$f(x) = \sqrt{x^2+y^2}$

if the tang. line is vertical.