MIDTERM 2

Math 251

Spring 2008

Name:

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{x^2+1}$

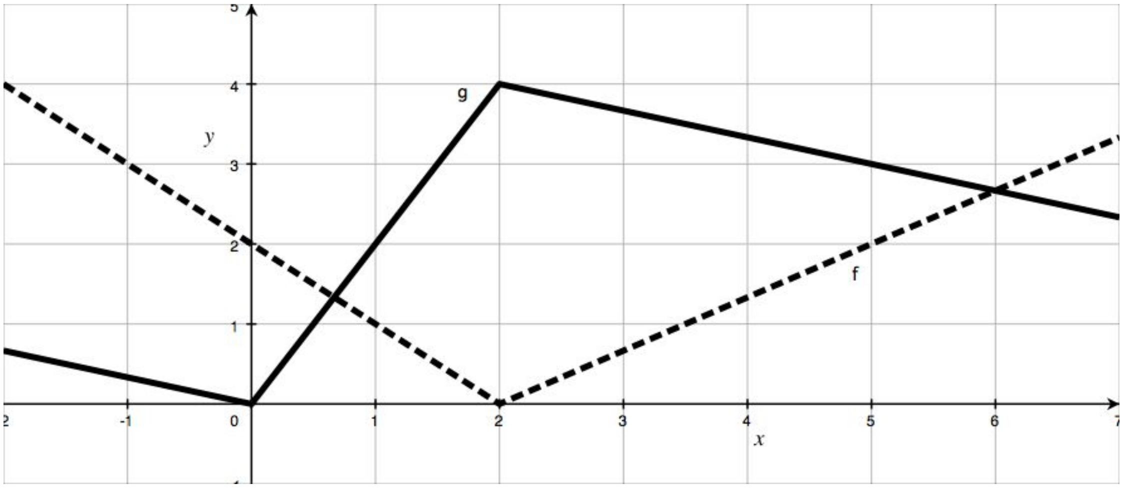
Show your work for the following problems. You need only simplify if the question explicitly asks for it.

1. [4] Use any results covered in class to find the following:

$$\lim_{x\to 0} \frac{\sin x}{x}$$

$$\lim_{x\to 0} \frac{3\sin(4x)}{2\sin(3x)}$$

2. [12] Let the graph of f and g be those shown below.



Define

$h(x) = 5f(x) - 4g(x)$

$j(x) = f(g(x))$

$u(x) = f(x)g(x)$

$v(x) = f(x)/g(x)$

Find the following:

$h'(1)$

$j'(5)$

$u'(1)$

$v'(5)$

3. [15] Find the derivatives of the following:

$g(x) = 2^{\log_2(\pi x)}$

$h(x) = \ln \frac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}$

$m(x) = \frac{1}{x}\arcsin(x)$

$n(x) = (\cos x)^x$

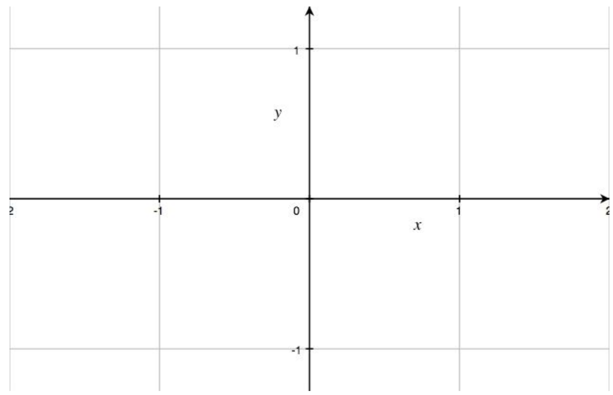
4. [6] *Prove* the following using only the definition of a derivative and limit properties. Clearly explain each of your steps.

   If c is a real number and f is a differentiable function for all x, then

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x)$$

5. Consider the relation described by $x^2 + y^2 = 1$.

(a) [2] Draw the collection of ordered pairs (x, y) that satisfy the relation $x^2 + y^2 = 1$.



(b) [1] On the graph above, draw the line tangent to the graph of $x^2 + y^2 = 1$ at $(\cos \frac{-\pi}{4}, \sin \frac{-\pi}{4})$.

(c) [7] Find the equation of the tangent line you just drew.

(d) [3] Let θ be the measure of an angle between 0 and 2π starting from the positive x-axis. Give the coordinates where the terminal side intersects the graph in terms of θ.

(e) (Extra Credit!) Find the equation of the line that is tangent to the graph of $x^2 + y^2 = 1$ at the coordinates you just wrote down for (d).