MIDTERM 1 **Math 251** Spring 2008

Name:

1. [2] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function.

   T F $(x+y)^{-2} = \sqrt{x+y}$

   T F $\lim_{x \to a} f(x) = f(\lim_{x \to a} x)$

   T F If $f'(r)$ exists, then $\lim_{x \to r} f(x) = f(r)$.

   T F The absolute value function is a differentiable function.

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

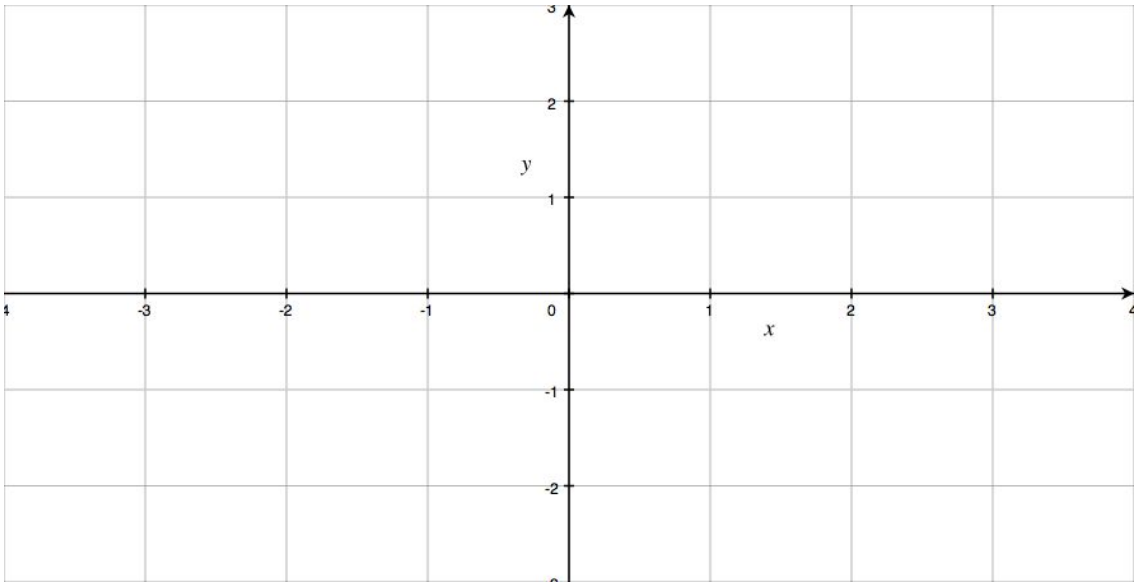
2. [2] If the tangent line to $y = f(x)$ at $(4, 3)$ passes through the point $(0, 2)$ find the following.

   (a) $f(4)$

   (b) $f'(4)$

3. [14] Given the rules of f and g below, graph both functions on the axis provided and evaluate the following (if they exist!):

$$f(x) = \begin{cases} (x+2)^2 - 1 & \text{if } x \leq 0, \\ -2x + 2 & \text{if } 0 < x < 3, \end{cases} \qquad g(x) = 2\sin(\tfrac{\pi}{4}x)$$



$\lim_{x \to 0-} f(x)$ $\qquad$ $\lim_{x \to -\infty} g(x)$

$f'(2)$ $\qquad$ $\lim_{x \to -2}[6g(x) - f(x)]$

Make a rough sketch of the graph of $f'(x)$:

4. [12] Find the limit if it exists, or explain why it does not exist.

$\lim_{x \to -1} (3x^4 + 2x^2 - x + 1)$ $\lim_{x \to 5^+} \ln(x - 5)$

$\lim_{x \to \infty} \arctan \frac{x^2 - 7}{x^5 - 4x + 8}$ $\lim_{x \to 0} x^2 \sin \frac{\pi}{x}$

5. [4] Is there a number that is exactly 1 more than its cube? Justify your answer.

6. [5] Let $f(x) = x^2 - e$, where e is approximately 2.718. Find the equation for the line tangent to the graph of f, when $x = 1$.

7. If a rock is thrown upward on the planet Mars with a velocity of 10m/s, its height (in meters) after t seconds is given by $H(t) = 10t - 2t^2$.

   (a) [5] Find a function that describes the instantaneous velocity of the ball after t seconds using only methods introduced in class thus far.

   (b) [3] When does the ball reach its highest point?

   (c) [3] When does the rock hit the surface?