MIDTERM 1 **Math 251** Spring 2008

Name: KEY

1. [2] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f be a function.

   T F $(x+y)^{-2} = \sqrt{x+y}$ $(x+y)^{-2} = \frac{1}{(x+y)^2}$

   T F $\lim_{x\to a} f(x) = f(\lim_{x\to a} x)$ if f is not cont.

   T F If $f'(r)$ exists, then $\lim_{x\to r} f(x) = f(r)$.
   differentiable cont

   T F The absolute value function is a differentiable function. there is a corner at $x=0$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [2] If the tangent line to $y = f(x)$ at $(4,3)$ passes through the point $(0,2)$ find the following.

   (a) $f(4) = 3$ (+1)

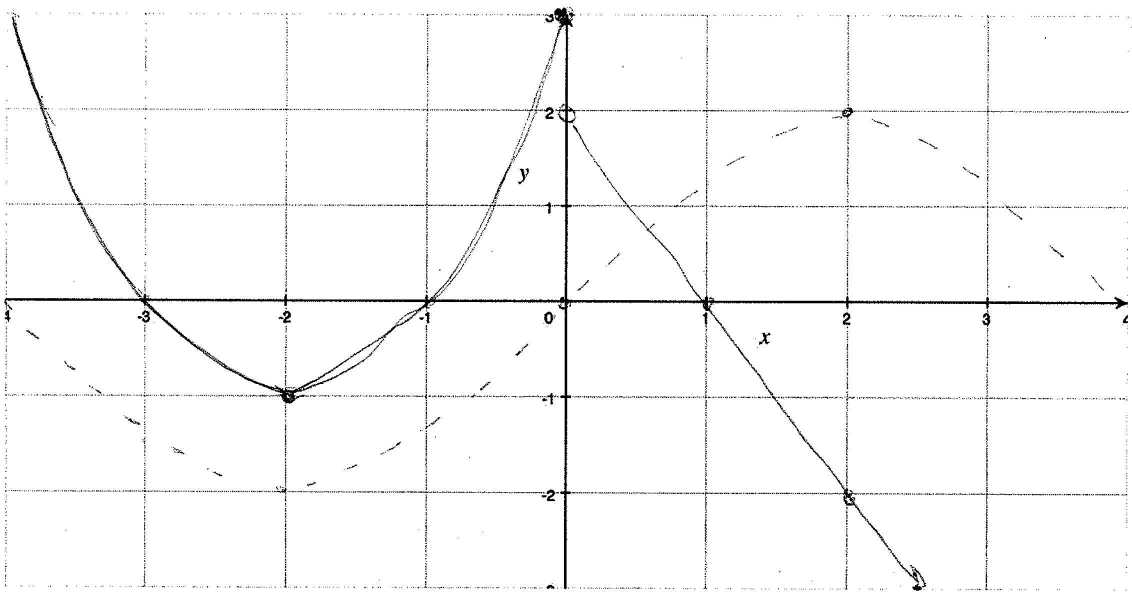
   (b) $f'(4) = \frac{3-2}{4-0} = \frac{1}{4}$ (+1)

3. [14] Given the rules of f and g below, graph both functions on the axis provided and evaluate the following (if they exist!):

$$f(x) = \begin{cases} (x+2)^2 - 1 & \text{if } x \le 0, \\ -2x+2 & \text{if } 0 < x < 3, \end{cases} \qquad g(x) = 2\sin(\tfrac{\pi}{4}x)$$

graph of f (+2)

graph of g (+2)



$\lim_{x \to 0-} f(x)$

3 (+2)

$\lim_{x \to -\infty} g(x)$

(+2) DNE

$f'(2)$

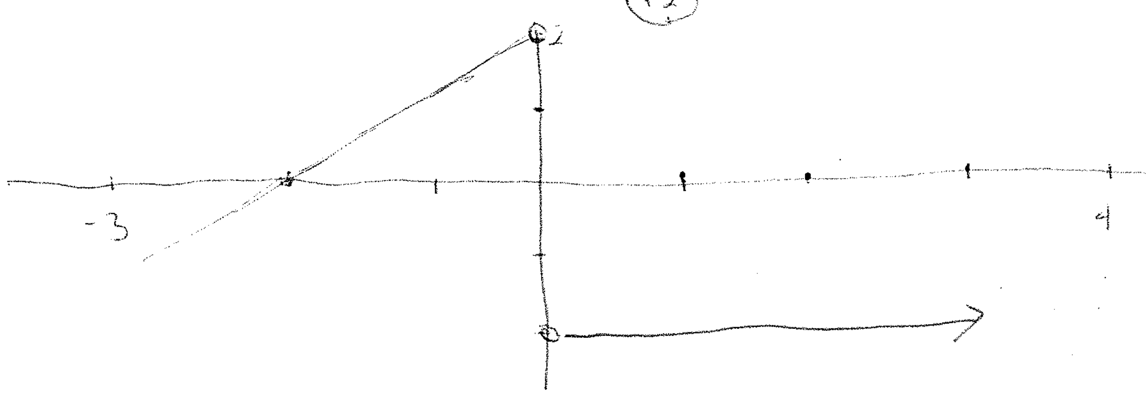
-2 (+2)

$\lim_{x \to -2} [6g(x) - f(x)]$

$6 \lim_{x \to -2} g(x) - \lim_{x \to -2} f(x)$ (+2)

$6(-2) - -1 = -12 + 1 = -11$

Make a rough sketch of the graph of $f'(x)$: (+2)



4. [12] Find the limit if it exists, or explain why it does not exist.

$$\lim_{x \to -1} (3x^4 + 2x^2 - x + 1)$$

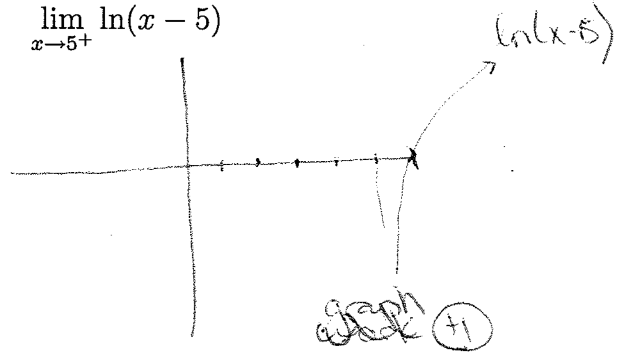
$= 3(-1)^4 + 2(-1)^2 - (-1) + 1$ (+1)

$= 3 + 2 + 1 + 1$

$= 7$ (+1)

notation (+1)

$$\lim_{x \to 5^+} \ln(x - 5)$$



$-\infty$ (+1)

graph ~~work~~ (+1)

~~notation~~ (+1)
ln(0) DNE

$$\lim_{x \to \infty} \arctan \frac{x^2 - 7}{x^5 - 4x + 8}$$

$= \arctan \lim_{x \to \infty} \frac{x^2-7}{x^5-4x+8}$

note $\lim_{x \to \infty} \frac{x^2-7}{x^5-4x+8} \frac{(1/x^5)}{(1/x^5)}$ inside (+2)

$= \lim_{x \to \infty} \frac{\frac{1}{x^3} - \frac{7}{x^5}}{1 - 4/x^4 + 8/x^5}$

$= 0$

$\Rightarrow \arctan \lim_{x \to \infty} \frac{x^2-7}{x^5-4x+8}$

$= \arctan 0 = 0$ outside (+1)

$$\lim_{x \to 0} x^2 \sin \frac{\pi}{x}$$

note $-1 \le \sin \pi/x \le 1$

since $x^2 > 0$ for all x

$\Rightarrow -x^2 \le x^2 \sin \pi/x \le x^2$

Note also $\lim_{x \to 0} -x^2 = 0 = \lim_{x \to 0} x^2$

So the squeeze thm $\Rightarrow \lim_{x \to 0} x^2 \sin \pi/x = 0$

looked for squeeze (+1)

~~Sketched~~ used correctly (+1)

sense (+1)

5. [4] Is there a number that is exactly 1 more than its cube? Justify your answer.

y up.

(+2) We are looking for a number x such that
$x = x^3 + 1 \Rightarrow x^3 - x + 1 = 0$.

(+1) If we show the graph of $y = x^3 - x + 1$ has a root this will imply such an x exists.

Knows to try IVT (+1)

Notice if $x = -2$ then $y = -8 + 2 + 1 = -5 < 0$
and if $x = 2$ then $y = 8 - 2 + 1 = 7 > 0$.
Since $x^3 - x + 1$ is a cont function the intermediate value thm $\Rightarrow$ such an x exists.

6. [5] Let $f(x) = x^2 - e$, where e is approximately 2.718. Find the equation for the line tangent to the graph of f, when $x = 1$.

$$f'(x) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{((1+h)^2 - e) - (1^2 - e)}{h}$$

(+1)

$$= \lim_{h \to 0} \frac{\cancel{1} + 2h + h^2 \cancel{- e} - \cancel{1} + \cancel{e}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2+h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2 + h = 2$$ (+1)

looking for a line (+1)
notation (+1)

$\Rightarrow$ slope of the line we're looking for is 2.

$y = mx + b$
$\Rightarrow y = 2x + b$

Since the point $(1, 1-e)$ is on the line as well we have: (+1)

$$1 - e = 2(1) + b$$
$$-1 - e = b$$

Thus the eq of the line is $y = 2x - 1 - e$

7. If a rock is thrown upward on the planet Mars with a velocity of 10m/s, its height (in meters) after t seconds is given by $H(t) = 10t - 2t^2$.

(a) [5] Find a function that describes the instantaneous velocity of the ball after t seconds using only methods introduced in class thus far.

inst velocity at time $t = H'(t)$

$= \lim_{h \to 0} \frac{H(t+h) - H(t)}{h} = \lim_{h \to 0} \frac{(10(t+h) - 2(t+h)^2) - (10t - 2t^2)}{h}$

looking for der (+1)

plugged in right (+1)

notation (+1)

alg (+1)

$= \lim_{h \to 0} \frac{\cancel{10t} + 10h - 2(t^2 + 2th + h^2) - \cancel{10t} + 2t^2}{h}$

$= \lim_{h \to 0} \frac{10h - \cancel{2t^2} - 4th - 2h^2 + \cancel{2t^2}}{h} = \lim_{h \to 0} \frac{\cancel{h}(10 - 4t - 2h)}{\cancel{h}}$

$= 10 - 4t$ (+1)

(b) [3] When does the ball reach its highest point?

occurs when $H'(t) = 0$ (+2)

so @ $0 = 10 - 4t$

$\Rightarrow +10 = +4t$

$\frac{5}{2} = \frac{10}{4} = t$

after $5/2$ sec (+1)

(c) [3] When does the rock hit the surface?

occurs when $H(t) = 0$ (+1)

$0 = 10t - 2t^2 = t(10 - 2t)$

solving correctly (+1)

$\Rightarrow t = 0$ or $10 - 2t = 0$

$\Rightarrow 2t = 10$

$t = 5$ sec (+1)