Mean Value Theorem $\S4.2$

Get into a group of three people and work on the following problems. Only turn in *ONE* copy from each group by Tuesday the 27th by 6pm. Make sure that your answers are written up completely and clearly (with correct notation!!!) as there will be no opportunity for rewriting problems.

- 1. Consider the function $f(x) = \cos 2x$ with a domain of $[\pi/8, 7\pi/8]$.
 - (a) State Rolle's Theorem.
 - (b) Verify the three hypotheses of Rolle's Theorem.
 - (c) Find all numbers c that satisfy the conclusion of Rolle's Theorem.

- 2. By completing the following steps we will show $f(x) = x^3 + x 1$ has only one real root.
 - (a) State the Intermediate Value Theorem.
 - (b) Use the Intermediate Value Theorem to show that f has at least one real root.
 - (c) Assume that there are at least two real roots. You need *not* find them, but let us say that that f has a root at x = a and x = b. What is f(a) and f(b) equal to?
 - (d) State Rolle's Theorem.
 - (e) Use Rolle's Theorem to show that f' must have a root.
 - (f) Find the rule for f'(x). Does f' have any roots?
 - (g) Given that your answers for part (f) and part (e) cannot both be true, the assumptions you made to arrive at part (e)'s conclusion must have been false. Recall the assumption you made in part (c) that lead you to the conclusions in part (e). This assumption must have been false. What assumption was it, and since it is false, what can you conclude?