§4.5 #13

Graphing Functions §4.5

Get into a group of three people and work on the following problems. Only turn in *ONE* copy from each group by Friday the 6th by 6pm. Make sure that your answers are written up completely and clearly (with correct notation!!!) as there will be no opportunity for rewriting problems.

1. Graph $y = \frac{x}{x^2+9} = f(x)$

Domain: Note $x^2+9 \neq 0 \Rightarrow$ domain is $\mathbb{R}$

x int: $0 = \frac{x}{x^2+9} \Rightarrow x = 0$ $(0,0)$

y int: $\frac{0}{0^2+9} = \frac{0}{9} \Rightarrow y = 0$ $(0,0)$

Symmetry $f(-x) = \frac{-x}{(-x)^2+9} = -\frac{x}{x^2+9} = -f(x)$ odd

HA $\lim_{x\to\infty} \frac{x}{x^2+9} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1+\frac{9}{x^2}} = \frac{0}{1+0} = 0$

note sym $\Rightarrow \lim_{x\to-\infty} f(x) = 0$

V.A: no canidates that I see

$f'(x) = \frac{(x^2+9)\cdot 1 - x(2x)}{(x^2+9)^2} = \frac{9-x^2}{(x^2+9)^2} = \frac{(3-x)(3+x)}{(x^2+9)^2}$

Critical pts • $f'(x)$ has a domain of $\mathbb{R}$

• $f'(x) = 0 = \frac{9-x^2}{(x^2+9)^2} \Rightarrow 9-x^2=0$

$\Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

$f'(x)$: $\frac{-}{+}$ | $\frac{+}{+}$ | $\frac{-}{+}$

dec -3 inc 3 dec

local min $(-3, \frac{-3}{18})$ local max $(3, \frac{1}{6})$

$f''(x) = \frac{(x^2+9)^2\cdot(-2x) - (9-x^2)2(x^2+9)2x}{[(x^2+9)^2]^2}$

$= \frac{-2x^3-18x-(36x-4x^3)}{(x^2+9)^3}$

$= \frac{+2x^3-54x}{(x^2+9)^3} = \frac{+2x(x^2-27)}{(x^2+9)^3}$

possible pts of inflection

$0 = f''(x) = \frac{+2x(x^2-27)}{(x^2+9)^3}$

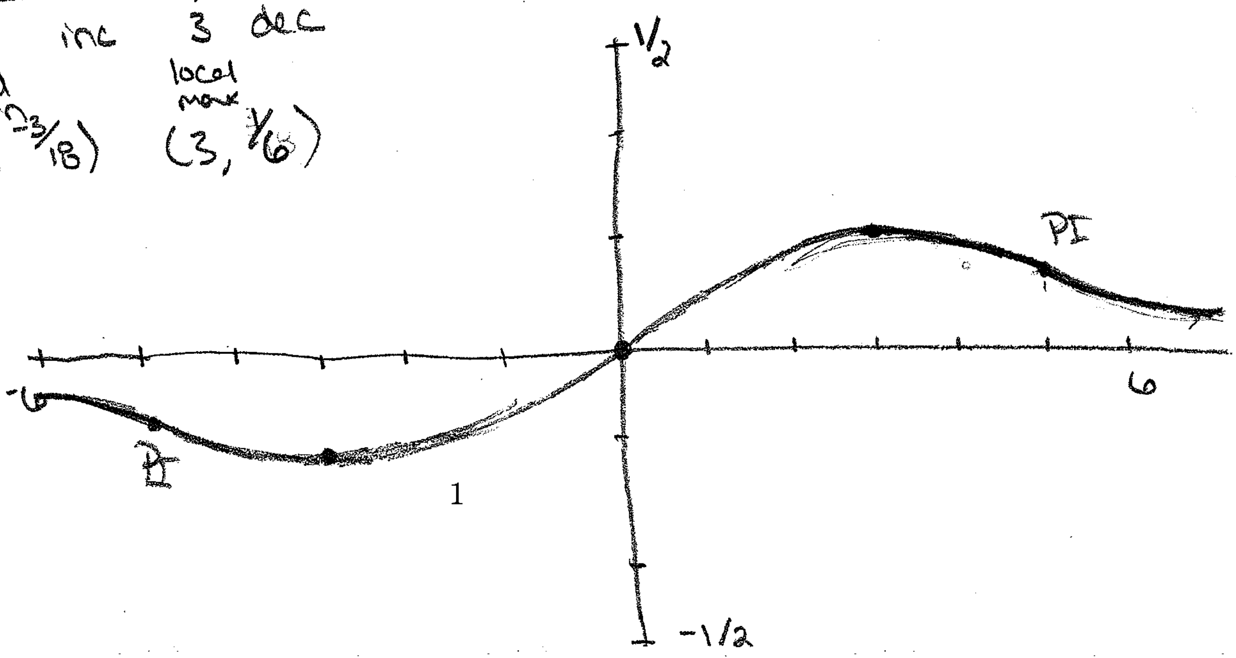
$\Rightarrow +2x = 0$ or $x^2-27=0$

$x = 0$ or $x = \pm\sqrt{27} \approx 5.1$

$\frac{-\cdot+}{+}$ | $\frac{-\cdot-}{+}$ | $\frac{+\cdot-}{+}$ | $\frac{+\cdot+}{+}$

CD $-\sqrt{27}$ CU 0 CD $\sqrt{27}$ CU

$(-\sqrt{27}, \frac{-\sqrt{27}}{27+9})$ $(\sqrt{27}, \frac{\sqrt{27}}{36})$



#39 §4.5

2. Graph $y = e^{\sin x}$ $= g(x)$

Domain $\mathbb{R}$

X-int $0 = e^{\sin x}$ $\Rightarrow$ there is no x-intercept

Y-int $e^{\sin 0} = e^0 = 1$ $(0,1)$

Symmetry $g(-x) = e^{\sin(-x)} = e^{-\sin x}$ b/c sin is an odd funct

$\neq e^{\sin x}$ or $-e^{\sin x}$ not even or odd

~~HA sin~~ $g(x+2\pi) = e^{\sin(x+2\pi)} = e^{\sin x} = g(x)$
periodic?

HA $\lim_{x\to\infty} e^{\sin x}$ DNE b/c $\sin x$ is periodic

VA no den. are misbehaving so

$g'(x) = e^{\sin x} \cdot \cos x$

Critical points • $g'(x)$ is defined on $\mathbb{R}$

• $g'(x) = 0 = e^{\sin x} \cdot \cos x$

$e^{\sin x} \neq 0$ but $\cos x = 0$

when $x = \frac{\pi}{2} + 2\pi k$

$= \frac{3\pi}{2} + 2\pi k$

$g'(x)$

0	$+\cdot+$	$\frac{\pi}{2}$	$+\cdot-$	$\frac{3\pi}{2}$	$+\cdot+$	2π
	inc	local max $(\frac{\pi}{2}, e^1)$	dec	local min $(\frac{3\pi}{2}, \frac{1}{e})$	inc	

$g''(x) = e^{\sin x}(-\sin x) + e^{\sin x}\cos^2 x$

$= e^{\sin x}(\cos^2 x - \sin x)$

possible pts of inflect

$0 = e^{\sin x}(\cos^2 x - \sin x)$

$e^{\sin x} \neq 0$ $\cos^2 x - \sin x = 0$

recall $\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

So

$\cos^2 x - \sin x = 0$

$1 - \sin^2 x - \sin x = 0$

$-\sin^2 x - \sin x + 1 = 0$

let $u = \sin x$

$-u^2 - u + 1 = 0$

$\Rightarrow u = \frac{1 \pm \sqrt{1+4}}{-2} = \frac{1 \pm \sqrt{5}}{-2}$

$\Rightarrow \sin x = \frac{1+\sqrt{5}}{-2}$ or $\frac{1-\sqrt{5}}{-2}$

$\frac{1+\sqrt{5}}{-2} < -1$ so $\sin x \neq \frac{1+\sqrt{5}}{-2}$

$x = \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right) \approx .67$

or $\pi - \sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right) \approx 2.4$

$\frac{1-\sqrt{5}}{2}$

note $e^{\sin x}$ is always positive so we need only worry about the parabola

$\frac{1+\sqrt{5}}{-2}$ $\frac{1-\sqrt{5}}{-2}$

$g''(x)$

0	$+$	$.67$	$-$	2.4	$+$	2π
	CU	$(.67, e^{\frac{\sqrt{5}-1}{2}})$	CD	$(2.4, e^{\frac{\sqrt{5}-1}{2}})$	CU