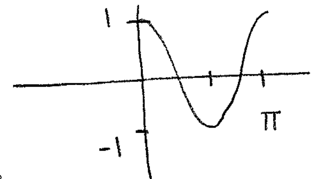
Mean Value Theorem §4.2

Get into a group of three people and work on the following problems. Only turn in *ONE* copy from each group by Tuesday the 27th by 6pm. Make sure that your answers are written up completely and clearly (with correct notation!!!) as there will be no opportunity for rewriting problems.

1. Consider the function $f(x) = \cos 2x$ with a domain of $[\pi/8, 7\pi/8]$.



   (a) State Rolle's Theorem.

   Let F be a function so that
   1) F is con't on $[a,b]$
   2) F is diff. on (a,b)
   3) $F(a) = F(b)$

   Then there exists a c between a & b so that $F'(c) = 0$.

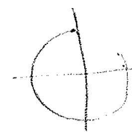
   (b) Verify the three hypotheses of Rolle's Theorem.

   If $F(x) = f(x) = \cos 2x$

   ✓ 1) we know $2x$ is con't & $\cos x$ is con't & (in) ⇒ $\cos 2x$ is con't on $[\pi/8, 7\pi/8]$

   ✓ 2) $F'(x) = -2\sin 2x$ ⇒ F is dif on $(\pi/8, 7\pi/8)$

   ✓ 3) $F(\pi/8) = \cos \pi/4 = 1/\sqrt{2} = 1/\sqrt{2} = \cos 7\pi/4 = F(7\pi/8)$

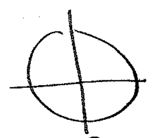


   (c) Find all numbers c that satisfy the conclusion of Rolle's Theorem.

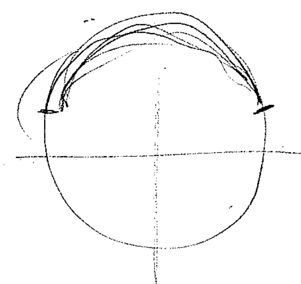
   We are looking for c so that

   $-2\sin 2c = f'(c) = 0$

   ⇒ $\sin 2c = 0$



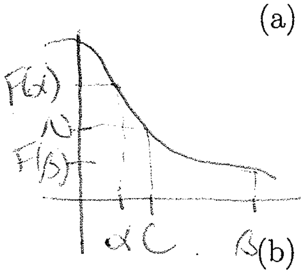
   ⇒ $2c = 0 + 2\pi k$ or $\pi + 2\pi k$ for integers k



   ⇒ $c = \pi k$ or $\pi/2 + \pi k$

   ⇒ $c = \pi/2$ to stay between $\pi/8$ & $7\pi/8$

2. By completing the following steps we will show $f(x) = x^3 + x - 1$ has only one real root.



(a) State the Intermediate Value Theorem.

If F is a con't. function on $[\alpha, \beta]$ and N is a number between $F(\alpha)$ and $F(\beta)$ then there exists a # c between α & β so that $F(c) = N$

(b) Use the Intermediate Value Theorem to show that f has *at least* one real root.

From ch 2 we know f is con't on $\mathbb{R}$
0 is between $f(-100) < 0$ and $f(100) > 0$
so by IVT there exists a c between -100 & 100 so that $f(c) = 0$

(c) Assume that there are at least two real roots. You need *not* find them, but let us say that that f has a root at $x = a$ and $x = b$. What is $f(a)$ and $f(b)$ equal to?

$f(a) = 0 = f(b)$

(d) State Rolle's Theorem.

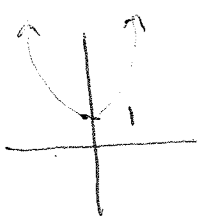
If G is a function such that
1) G is con't on $[a,b]$
2) G is diff. on (a,b)
3) $G(a) = G(b)$

Then there exists a γ between a & b so that $G'(\gamma) = 0$

(e) Use Rolle's Theorem to show that f' must have a root.

If we consider f
1) f is con't on $\mathbb{R}$ so also on $[a,b]$
2) f is diff on $\mathbb{R}$ so also on (a,b)
3) $f(a) = f(b)$ by part c
Rolle's Thm => there exists a root of f'. (a γ between a & b so $f'(\gamma) = 0$)

(f) Find the rule for $f'(x)$. Does f' have any roots?



$f'(x) = 3x^2 + 1$ no. parabola opening upwards shifted up 1 unit

(g) Given that your answers for part (f) and part (e) cannot both be true, the assumptions you made to arrive at part (e)'s conclusion must have been false. Recall the assumption you made in part (c) that lead you to the conclusions in part (e). This assumption must have been false. What assumption was it, and since it is false, what can you conclude?

We assumed that there where 2 roots
There must only be 1 root