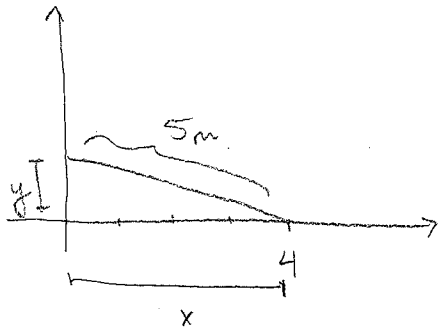
KEY

# Related Rates §3.9

Get into a group of three people and work on the following problems. You are welcome to use the book and your notes as a reference. Only turn in *ONE* copy from each group by Monday the 19th by 6pm. Make sure that your answers are written up completely and clearly (with correct notation!!!) as there will be no opportunity for rewriting problems.

1. A five meter ladder rests against a vertical wall until the bottom of the ladder slides away from the wall at a rate of 1m/s.

   (a) Draw a picture of the situation when the bottom of the ladder is 4 meters away from the wall.




   (b) How fast is the bottom of the ladder moving away from the wall at the moment you just drew?

1 m/s

   (c) Find how fast the top of the ladder is sliding down the wall at the moment you drew in part (a)?

from class

$\frac{dy}{dt} = \frac{-x}{\sqrt{25-x^2}}$

$\Rightarrow \left.\frac{dy}{dt}\right|_{x=4} = \frac{-4}{\sqrt{25-16}} = \frac{-4}{\sqrt{9}} = \frac{-4}{3}$

OR

$x^2+y^2 = 5^2$

$\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ b/c $\frac{dx}{dt} = 1$

$\Rightarrow 2x + 2y\frac{dy}{dt} = 0$

$\Rightarrow \frac{dy}{dt} = \frac{-2x}{2y} = \frac{-x}{y}$

So $\left.\frac{dy}{dt}\right|_{x=4} = -4/3$

   (d) Explain the sign in your answer for part (c).

the ladder is moving downwards.

2. Air is being pumped into a spherical balloon so that is volume increases at a rate of 100 $cm^3/s$. How fast is the radius of the balloon increasing when the diameter is 50 cm?

let $V$ be volume of the balloon

$\frac{dV}{dt} = 100\ cm^3/s$ we want $\left.\frac{dr}{dt}\right|_{diam=50}$

we know volume of the balloon is $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = \frac{4\pi}{3}\frac{d}{dt}(r^3)$$

$$= \frac{4\pi}{3}\, 3r^2 \frac{dr}{dt}$$

consider ex 1 pg 241

so at diam = 50, $r = 25$

$$\Rightarrow 100 = \left.\frac{dV}{dt}\right|_{diam=50} = \frac{4\pi}{3}\cdot 3(25)^2 \cdot \left.\frac{dr}{dt}\right|_{r=25}$$

$$\Rightarrow 100 \cdot \frac{3}{4\pi \cdot 3 \cdot 25^2} = \left.\frac{dr}{dt}\right|_{r=25}$$

$$\frac{25 \cdot 4 \cdot 3}{4\pi \cdot 3 \cdot 25^2}$$

$$\frac{1}{25\pi}$$