

KEY

Rates of Change §3.7

Get into a group of three people and work on the following problems. You are welcome to use the book and your notes as a reference. Only turn in *ONE* copy from each group by Monday the 19th by 6pm. Make sure that your answers are written up completely and clearly (with correct notation!!!) as there will be no opportunity for rewriting problems.

1. The position of a particle is given by the equation: $s = f(t) = t^3 - 6t^2 + 9t$, where t is measure in seconds and s in meters.

- (a) Find the instantaneous velocity after 2 seconds.

$$f'(t)|_{t=2} = 3t^2 - 12t + 9 |_{t=2} = 3 \cdot 4 - 24 + 9 = 12 - 24 + 9$$

$$= -12 + 9 = -3 \text{ m/s} \quad (\text{backwards})$$

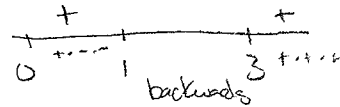
- (b) When is the particle moving forward (that is, in the positive direction)?

$$f'(t) = 3t^2 - 12t + 9 = 0$$

$$= 3(t^2 - 4t + 3) = 0$$

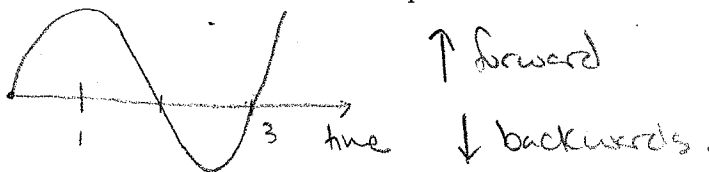
$$3(t-3)(t-1) = 0$$

$$\Rightarrow t = 3 \text{ or } 1$$



moving forward on the intervals $(0, 1)$ and $(3, \infty)$

- (c) Draw a diagram to represent the motion of the particle.



- (d) Find the total distance traveled by the particle during the first five seconds.

dist traveled from $t=0$ to $t=1$ + dist traveled from $t=1$ to $t=3$ + dist traveled from $t=3$ to $t=5$

$$|f(1) - f(0)| + |f(3) - f(1)| + |f(5) - f(3)|$$

$$|(1 - 6 + 9) - (0)| + |(27 - 6 \cdot 9 + 27) - (-5 + 9)| + |(125 - 6 \cdot 25 + 45) - (-4 - 54)|$$

$$|4| + |0 - 4| + |25(5 - 6) + 45 - 0|$$

$$4 + 4 + |-25 + 45|$$

$$8 + 20 = 28 \text{ m}$$

consider
at 1 on pg 201

2. Let $n = f(t)$ be the number of bacteria at time t . Note that

$$\text{average rate of growth} = \frac{\Delta n}{\Delta t} = \frac{f(t_2) - f(t_1)}{t_2 - t_1}$$

The instantaneous growth is obtained from the average rate of growth by letting the time period Δt approach 0:

$$\text{growth rate} = \lim_{\Delta t \rightarrow 0} \frac{\Delta n}{\Delta t} = \frac{dn}{dt}$$

- (a) Notice that most functions we can take the derivative of are continuous. Explain why most continuous functions won't accurately describe n .

population is discrete

- (b) Suppose we'd like to look at a large enough population that the comments you made above won't matter much. Suppose that by sampling the population at certain intervals it is determined that the population double every hour. That is, if the initial population is denoted by n_0 , then $f(1) = 2n_0$, and $f(2) = 2 * 2n_0$, and so forth. Find a continuous function that describes this observed behavior.

$$f(t) = 2^t n_0$$

- (c) Find the rate of growth of the bacteria at time t .

we want to find $f'(t)$

consider
ex 6
or
pg 226

$$\begin{aligned} f'(t) &= \frac{d}{dt}(n_0 2^t) \\ &= n_0 \frac{d}{dt}(2^t) \\ &= n_0 2^t \ln 2 \end{aligned}$$

since n_0 is a const.