KEY

Math 251
Logarithmic Differentiation §3.6

Get into a group of three people and work on the following problems. You are welcome to compare your answers with other groups but only after you have completed the problem. Also feel free to use the book and your notes as a reference. Only turn in *ONE* copy from each group by Monday the 12th by 6pm. Make sure that your answers are written up completely and clearly (with correct notation!!!) as there will be no opportunity for rewriting problems that you get wrong.

Let b be a positive real number. Recall the properties of logarithms:

$\log_b(xy) = \log_b x + \log_b y$

$\log_b(\frac{x}{y}) = \log_b x - \log_b y$

$\log_b(x^y) = y \log_b x$

Note: you need to *know* these for quizzes and exams as they will not be provided for you in the future!!!

1. Let $y = (\sin x)^{\ln x}$.

   - True or False: $\frac{dy}{dx} = (\ln x)(\sin x)^{\ln x - 1}$.

     FALSE!

   - Use logarithmic differentiation to find $\frac{dy}{dx}$.

$y = (\sin x)^{\ln x}$

$\Rightarrow \ln y = \ln[(\sin x)^{\ln x}]$

$= (\ln x)\ln(\sin x)$ by third. prop.

$\frac{d}{dx}(\ln y) = \frac{d}{dx}[\ln(x) \cdot \ln(\sin x)]$

$f(x) = \ln x \quad f'(x) = \frac{1}{x}$

$g(x) = y \quad g'(x) = dy/dx$

$\Rightarrow \frac{d}{dx}(\ln(y)) = f'(g(x))g'(x) = f'(y)\cdot\frac{dy}{dx} = \frac{1}{y}\frac{dy}{dx}$

$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(\ln(x)) \cdot \ln(\sin x) + \ln x \cdot \frac{d}{dx}(\ln(\sin x))$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} \cdot \ln(\sin x) + \ln x \frac{d}{dx}(\ln(\sin x))$

$f(x) = \ln x \quad f'(x) = \frac{1}{x}$

$g(x) = \sin x \quad g'(x) = \cos x$

$\frac{dy}{dx} = y\left(\frac{\ln(\sin x)}{x} + \ln x \cdot \frac{1}{\sin x} \cdot \cos x\right)$

$= (\sin x)^{\ln x}\left(\frac{\ln(\sin x)}{x} + (\ln x)\cdot \cot x\right)$

2. Let $y = x^{\sqrt{x}}$.

- True or False: $\frac{dy}{dx} = \sqrt{x}x^{\sqrt{x}-1}$.

FALSE?

- Use logarithmic differentiation to find $\frac{dy}{dx}$.

$\ln y = \ln x^{\sqrt{x}}$

$= \sqrt{x} \ln x$

$\downarrow \frac{d}{dx}$

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\sqrt{x} \ln x)$

$\frac{1}{y}\frac{dy}{dx} = \sqrt{x} \cdot \frac{d}{dx}(\ln x) + (\ln x)\frac{d}{dx}(\sqrt{x})$

$= \sqrt{x} \cdot \frac{1}{x} + (\ln x)\frac{1}{2}x^{-1/2}$

$\frac{dy}{dx} = y\left(\frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}}\right)$

or

$= x^{\sqrt{x}}\left(\frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}}\right)$

- Another method is to write $y = x^{\sqrt{x}} = (e^{\ln x})^{\sqrt{x}} = e^{\sqrt{x}\ln x}$. Use this to find $\frac{dy}{dx}$.

$y = e^{\sqrt{x}\ln x}$

$\downarrow \frac{d}{dx}$

$\frac{dy}{dx} = \frac{d}{dx}(e^{\sqrt{x}\ln x})$

$f(x) = e^x \quad f'(x) = e^x$

$g(x) = \sqrt{x}\ln x \quad g'(x) = \sqrt{x} \cdot \frac{1}{x} + (\ln x)\frac{1}{2}x^{-1/2}$

$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

$= e^{\sqrt{x}\ln x} \cdot \left(\frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}}\right)$

- Verify that your two answers are consistent.

note $e^{\sqrt{x}\ln x}\left(\frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}}\right) = e^{\ln x^{\sqrt{x}}}\left(\frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}}\right)$

$= x^{\sqrt{x}}\left(\frac{1}{\sqrt{x}} + \frac{\ln x}{2\sqrt{x}}\right)$ ✓