KEY

Math 251
The Chain Rule §3.4

Get into your assigned group of two or three people and work on the following problems. You are welcome to compare your answers with other groups but only after you have completed the problem. Also feel free to use the book and your notes as a reference. One copy from each group should be turned in by Wednesday at 6pm. Make sure that your answers are written up completely and clearly (with correct notation!!!) as there will be no opportunity for rewriting problems that you get wrong.

Recall the chain rule: If f and g are both differentiable and $F(x) = (f \circ g)(x) := f(g(x))$ then F is differentiable and

$$F'(x) = f'(g(x))g'(x)$$

For each of the problems below complete the following:

1. Define function f and g so that:
   - both functions are easily differentiable *and*
   - the given function F has the rule $f \circ g$ or equivalently $F(x) = f(g(x))$.
2. Find f' and g'.
3. Find $F'(x)$

(a) $F(x) = \sin x^2 = \sin(x^2)$

$f(x) = \sin x \qquad f'(x) = \cos x$

$g(x) = x^2 \qquad g'(x) = 2x$

$(f \circ g)(x) = f(g(x)) = f(x^2) = \sin(x^2) = F(x)$ ✓

$F'(x) = f'(g(x))\, g'(x)$

$= f'(x^2) \cdot 2x$

$= (\cos x^2)\, 2x$

(b) $F(x) = (x^3 - 1)^{100}$

$f(x) = x^{100} \qquad f'(x) = 100x^{99}$

$g(x) = x^3 - 1 \qquad g'(x) = 3x^2$

$(f \circ g)(x) = f(g(x)) = f(x^3 - 1) = (x^3 - 1)^{100} = F(x)$ ✓

$F'(x) = f'(g(x))\, g'(x)$

$= f'(x^3 - 1) \cdot 3x^2$

$= 100(x^3 - 1)^{99} \cdot 3x^2$

$= 300x^2(x^3 - 1)^{99}$

(c) $F(x) = e^{x^5}$

$f(x) = e^x \qquad f'(x) = e^x$

$g(x) = x^5 \qquad g'(x) = 5x^4$

$(f \circ g)(x) = f(g(x)) = f(x^5) = e^{x^5} = F(x)$ ✓

$F'(x) = f'(g(x)) \cdot g'(x)$

$= f'(x^5) \cdot 5x^4$

$= e^{x^5} \cdot 5x^4$

4. Notice that we can use the chain rule in conjunction with the previous rules we already learned. Use the work you did on the previous page to find $F'(x)$ given F as the following:

(a) $2^x + \sin x^2$

$$\frac{d}{dx}(2^x + \sin x^2) = \frac{d}{dx}2^x + \frac{d}{dx}(\sin x^2)$$
$$= 2^x \ln 2 + 2x \cos x^2$$

(b) $e^x(x^3 - 1)^{100}$

$$\frac{d}{dx}(e^x(x^3-1)^{100}) = \frac{d}{dx}(e^x)\cdot(x^3-1)^{100} + e^x \cdot \frac{d}{dx}(x^3-1)^{100}$$

Product Rule?

$$= e^x(x^3-1)^{100} + e^x 300x^2(x^3-1)^{99}$$

(c) $\frac{e^{x^4}}{\sqrt{x}}$

$$\frac{d}{dx}\left(\frac{e^{x^4}}{\sqrt{x}}\right) = \frac{\sqrt{x}\,\frac{d}{dx}(e^{x^4}) - e^{x^4}\frac{d}{dx}(\sqrt{x})}{(\sqrt{x})^2}$$

quotient rule?

note: we computed $\frac{d}{dx}(e^{x^5})$ on the front & not $\frac{d}{dx}(e^{x^4})$?

$f(x) = e^x \quad f'(x) = e^x$

$g(x) = x^4 \quad g'(x) = 4x^3$

$(f \circ g)(x) = f(g(x)) = f(x^4) = e^{x^4}$ ✓

$$\frac{d}{dx}(e^{x^4}) = f'(g(x))\cdot g'(x) = f'(x^4)\cdot 4x^3 = e^{x^4}\cdot 4x^3 = 4x^3 e^{x^4}$$

returning to $\frac{d}{dx}\left(\frac{e^{x^4}}{\sqrt{x}}\right) = \frac{\sqrt{x}\,\frac{d}{dx}(e^{x^4}) - e^{x^4}\frac{d}{dx}(\sqrt{x})}{(\sqrt{x})^2}$

$$= \frac{(\sqrt{x})4x^3e^{x^4} - e^{x^4}\frac{d}{dx}(x^{1/2})}{x}$$

$$= \frac{4e^{x^4}x^{7/2} - e^{x^4}\frac{1}{2}x^{-1/2}}{x}$$