Math 213

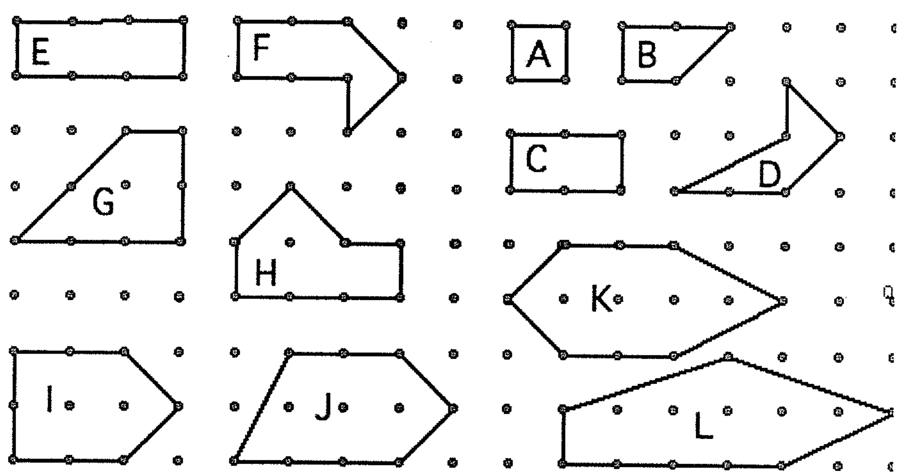
Areas on a Geoboard

adapted slightly from Scott Fallstrom's material

Key

Form yourselves into groups of two to answer the following questions. Turn in one copy for each group with all the group member's names on it. This worksheet is due Thursday 9/3 by 4:30 pm in my box.

There are three ways to find the areas on a Geoboard: 1) count squares and estimate the fractional pieces; 2) break the shape into triangles or other shapes and then add up the areas; 3) create a rectangle (or other shape whose area you can compute) around your shape, and then find the area inside the large shape but outside the original and subtract it from the area of the larger outer shape. An example of each type is given in your text on page 751. Use either techniques 2 or 3 to fill in the table below.



Shape	# of points on the interior	# of points on the border	Area
A	0	4	1
B	0	5	1.5
C	0	6	2
D	0	6	2
E	0	8	3
F	0	8	3
G	1	8	4
H	1	8	4
I	2	8	5
J	3	8	6
K	4	8	7
L	5	8	8

#1

Use the information from the previous page to answer the following questions:

1. If we add one point to the boundary/border and leave the number of interior points the same, what happens to the area of the whole shape?

(+1) the area is increased by 1/2 units²

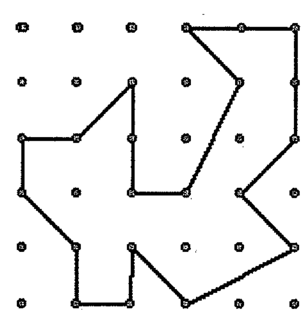
2. If we add one point to the interior by leave the number of border points the same, what happens to the area of the whole shape?

(+1) the area is increased by 1 units²

Use this knowledge to fill out the following table:

Explanation	Interior	Boundary	Area
Shape L from previous page	5	8	8
Add 1 to the boundary	5	9	8.5
Add 1 to the boundary	5	10	9
Add 1 to the interior	6	10	10
Add "I" to the interior	5+I	8	8+I
Add "B" to the boundary	5	8+B	8+B/2

What is the formula you end up with for the area of a polygon that is created on a rectangular grid?

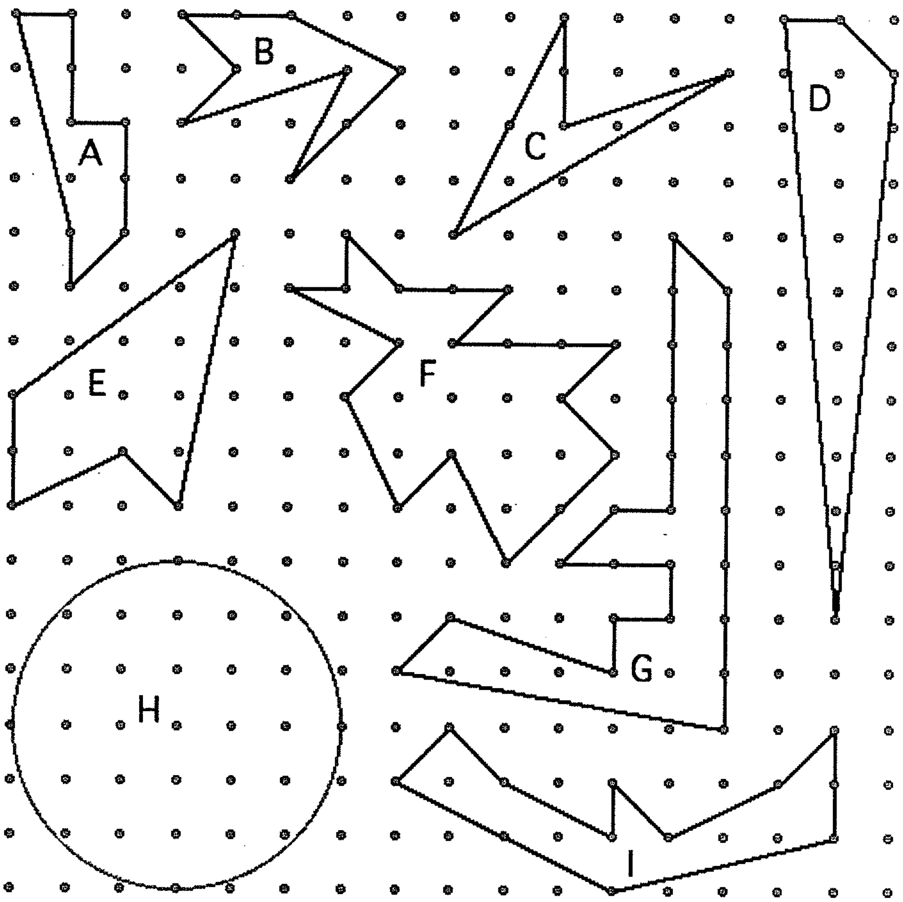


(+2) $\frac{B-2}{2} + I$

Use you formula to find the area of the above shape:

(+1) B = 20, I = 4 so $\frac{20-2}{2} + 4 = \frac{18}{2} + 4 = 9 + 4 = 13$

Use the formula you discovered at the bottom of page two to find the areas of the following shapes.



Shape	Area (units²)	Shape	Area (units²)
A	4.5	F $\frac{16}{2}+7$	15
B	4.5	G	15
C	4	H	26
D	11	I	9
E	16		

Did the formula fail on any shapes?

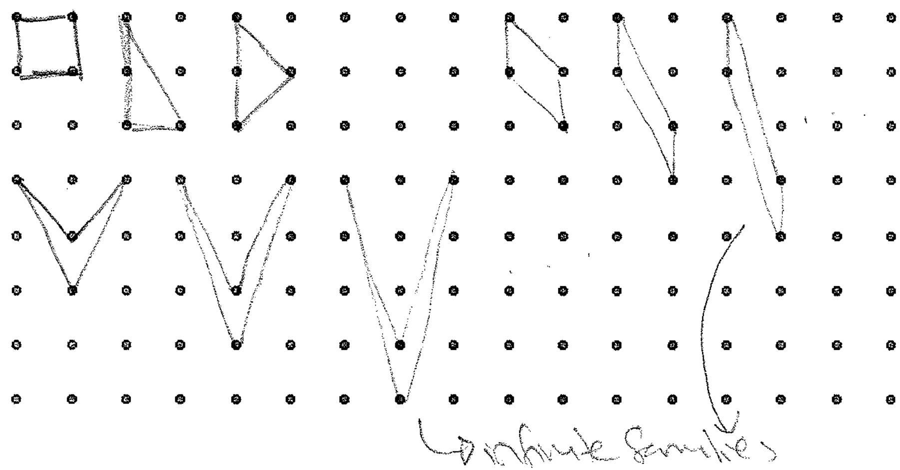
yes on H.

What shapes will this formula work with?

n-gons

1. Of the many shapes that have an area of 1 unit squared, there are only two possible values "B" and "I" can take, mainly "B"=4 or "B"=2 and "I"=1.

   (a) Draw some polygons with areas that are one unit squared.



   (b) How many polygons have an area of one and "B=4"? How many polygons have and area of one unit squared but "B=2" and "I"=1?

   +2

   There are infinitely many for B=4 but none with B=2 and I=1.

   (c) How many values can "B" and "I" take that will result in you being able to form a polygon with an area of 6 units squared?

   +1

   $\frac{B-2}{2} + I = 6$ → $B-2+2I=12$

   $\frac{B-2+2I}{2} = 6$ → $B+2I=14$

   So (B,I) pairs are: (14,0), (12,1), (10,2), (8,3), (6,4), (4,5)

2. Can you come up with a brief outline for an argument that would *justify* your observation made in number 2 on page two? (You are free to use the area formulas we discussed in class today.)

3. Can you come up with a brief outline for an argument that would *justify* your observation made in number 1 on page two? (You are free to use the area formulas we discussed in class today.)