Lab 4 $\qquad$ General Pythagoras

<div align="right">slightly adapted from Scott Fallstrom's material</div>

1. We will first set up a "Custom Tool" to create new shapes quickly. We'll be making squares and equilateral triangles. Start by making a line segment between two points (one on the left, one on the right). When finished, mark the left point as the center with a double click. Then select the line segment *and* the right point, and rotate them 90 degrees. This will form a right angle and the first two sides of a square. Now mark the newly rotated point as the center, select the line segment and the original left point, and rotate 90 degrees. Repeat until you are done creating the square. Once you are done creating the square, select the vertices clockwise in order. With these points still selected, choose the "Quadrilateral Interior" option from the Construct menu. This will color the square's interior.

2. Once you have the square shape with the interior marked, use the selection tool and click-and-drag across the entire square (including all vertices). Then press-and-hold the custom tool button (bottom of the toolbox) and choose "Create New Tool". Name the tool "Square". This tool will be available for us later.

3. Highlight all shapes on the screen and delete the objects (*No not make a new sketch*) Use the same idea to create an equilateral triangle. Start with a line segment (from 2 points), mark the left point as the center, and then select the right point and the line segment. Rotate by the angle needed and then connect the points with segments to finish the triangle. Once finished, select the vertices clockwise in order. With these points still selected, choose the 'Triangle Interior" option from the Construct menu. This will color the triangle's interior.

4. Once you have the triangular shape with the interior marked, use the selection tool and click-and-drag across the entire shape (including all the vertices). Then press-and-hold the custom tool button and choose "Create New Tool". Name the tool "Eq Triangle". This tool will be available for us later. Then highlight all the shapes on the screen and delete the object (*Do not make a new sketch*).

5. Now we will create a right triangle that will always remain a right triangle. Start with any two points A and B and create a line (not a line segment) between them. Mark one point as the center and then rotate *the line only* though 90 degrees. Select the newly rotated line and choose "Point on Line" from the Construct menu. Label this point C, and drag the point close to the original two points so that you can see all three points on the screen. Hide the original lines and create line segments between the three points. Label the points as you have before with the Display menu. Notice that if you click on any points, you'll be able to change the side lengths, but the triangle will remain a right triangle.

6. With your right triangle, click on each side (one by one) and use the "Length" option from the Measure menu to find the lengths of each side.

7. Click and hold on the custom tool button and choose "Square". Then select your vertices in order-one at a time. A square will form along the shape as you click and you want to make sure the square is on the *outside* of the triangle. Repeat along each set of vertices. When done, use the selection tool to click on the interior (shaded) region of each square and change the color so you have three different colors. Also, when you right lick on the colored region, you'll be able to select "Area" from the menu, do this for each square.

8. The hypotenuse of the right triangle has a square associated with it. Use the "Calculate" option under the Measure menu to find the sum of the areas of the other two squares. Move the points on the triangle around. What is always true about the area of the square on the hypotenuse?

9. While moving the points around, it is possible that the two smaller squares will have the same area (or really close to the same). What type of right triangle is required to make this happen? *Justify* yourself.

10. Now delete the squares and get back to the right triangle (you could also just delete all the shapes and repeat the previous steps to construct a right triangle). Choose the "Eq Triangle" tool and repeat what you did with the Square tool. This will mean that you find the area of the equilateral triangle associated with each side. Use "Calculate" again to find the sum of the areas of the two shapes that are *not* on the hypotenuse. Move the points around and then conjecture what is always true about the area of the equilateral triangle on the hypotenuse?

11. Make use of your observation in number 8 to *justify* your claim in 10.

12. Delete the equilateral triangles and get back to the right triangle (you could also just delete all the shapes and repeat the previous steps to construct a right triangle). Repeat what you did with the squares and equilateral triangles with circles centered at the midpoints of the sides of the right triangle and a radius that extends from the midpoint to the adjacent vertex of the right triangle. Use the "Calculate" to find the sum of the areas of the two circles that are *not* on the hypotenuse. Move the points around and then conjecture what is always true about the area of the circle centered at the midpoint of the hypotenuse with a radius extending from the midpoint to the nearest vertex.

13. Make use of you observation in number 8 to *justify* your claim in 12.

14. Delete all the previous shapes. Create *any* three points and form the line segments between them, Label the points A, B, and C. Once again, create the lengths of each side (with the Measure menu) but this time also form the measures of the angles. Move the length and the angle boxes so they correspond -the measure of angle B should be next to the length of $\overline{AC}$. As you move the points around, the angle measure and side lengths will change. There will always be a smallest angle, a middle angle, and a largest angle (of course, some of these could be congruent). Move the points around and then make a conjecture about the relationships between the lengths of the sides of the triangle and the measure of the angles.

15. Now with your triangle (that is probably not a right triangle), use the Square tool as before to create squares on each side length. As before, create the areas of each square. Compare the areas of the squares based on the shape of the triangle you form (move the points around to see differences, look for acute, right, and obtuse triangles)

    (a) If the area of the square on the longest side of the triangle is *larger* than the sum of the areas of the other two squares, then the triangle is a(n) _____ triangle.

    (b) If the area of the square on the longest side of the triangle is *smaller* than the sum of the areas of the other two squares, then the triangle is a(n) _____ triangle.

    (c) If the area of the square on the longest side of the triangle is *equal to* the sum of the areas of the other two squares, then the triangle is a(n) _____ triangle.

16. Use the above observations (not GSP) to determine what type of triangles are described below. Use *two terms* for each triangle from the following list: equilateral, scalene, isosceles, acute, right, obtuse.

Side lengths:	Type	Side lengths:	Type
4,5,6		4,4,9	
3,3,4		5,12,13	
3,3,3		5,5,8	
1,1,$\sqrt{2}$		5,6,10	