Lab 3

Several Triangle Centers

slightly adapted from Scott Fallstrom's material

We will consider a few points that are the "center" of a triangle in some sense. There are dozens of "centers" for a triangle, but here are a few:

1. Incenter (I): The center of the inscribed circles. This "center" is formed by the intersection of the angle bisectors.

2. Circumcenter (C): The center of the circumscribed circle. This "center" is formed by the intersection of the perpendicular bisectors.

3. Centroid (M): The intersection of the Medians of the triangle. A median is a line from the vertex to the midpoint of the opposite side.

4. Orthocenter (O): The intersection of the altitudes of the triangle. Remember an altitude is formed as the perpendicular line from a vertex to the side opposite.

5. Gergonne point (G): The intersection of the lines connecting the tangent point of the incircle to the opposite vertices.

6. Fermat point (F): This is also called the isogonic center or Rorricelli point. This point forms 120° angles with all the vertices.

We will be creating some of these centers and noting any properties we observe.

1. Centroid:

   (a) Start with any three points you want. Draw the line segments between them and make the segments "Thick". (You can use the Display menu or right-click on the line segment.) Label the triangle WXY. Select each line segment (no points) and choose the "Midpoints" option from the Construct menu. Form a thin line from each vertex to the midpoint opposite it. This will form 3 new lines. Select any two and choose the "Intersection" option from the Construct menu. This point is the Centroid of the triangle WXY. Click on each of the lines you just formed and hide them (Control H or Display menu). You could see the triangle WXY, the centroid, and the midpoints of the sides. Label the Centroid with an "M", the midpoint on $\overline{YW}$ as "A", the midpoint on $\overline{XW}$ as "B", and the midpoint on $\overline{XY}$ as "C".

   (b) Click on the point X, then on the point M. Choose the "Distance" option from the Measure menu. Repeat to find the distance MA. Do the same for the distance from each vertex of midpoint to M.

   (c) Click on any point on your triangle and move the point around. Do you see a relationship between XM and MA? What about between YM and MP? What about WM and MC? Don't worry about reporting or conjecturing just yet, just look around for a while.

   (d) Choose "Calculate" option from the Measure menu. Click on XM, click on the $\div$ symbol, then click on MA. This will form the ratio XM:MA. Repeat this for each vertex. What is the ratio you find?

   (e) Finish the following generalization of the above observation: "The centroid of a triangle will break the medians into lengths having a ratio of"

   (f) Click on the points XMY in order. From the Construct menu choose the "triangle interior" option. Repeat for WMY and WMX. You should have three triangles. Right click on each area to change the color so that the three triangles are different colors. You can highlight each of the colored triangles in turn and select Area from the Measure menu. Move the points on WXY around. What happens to the area of the colored triangles? What can you say about the centroid of any triangle.

2. Incenter:

   (a) Hide any information that is on your triangle WXY except for the points W, X, Y, and the centroid M. Selecting the vertices of the triangle in order, use the "Angle Bisector" option from the Construct menu to build the angle bisectors. Choose any two of the rays and use the "Intersection" from the Construct menu to find the incenter. Label your new point "I".

   (b) Verify that the point I is the incenter by creating a circle that touches each side exactly once. To do this, choose the point "I" and click on one line segment of the triangle WXY. From the Construct menu choose "perpendicular line". The distance from "I" to the intersection of the newly constructed perpendicular line with the side of the triangle is your radius.

   (c) Hide the perpendicular line, the intersection point you just constructed, and the circle you formed. You should be back to just the triangle WXY and the point I and M. Answer the following question by moving the points X, Y, or W. You will need to create the lengths XY, XW, and WY, as well as the angles across from them to see what type of triangles you actually have (acute, obtuse, equiangular, right).

      i. Are M and I always in the interior of WXY?

      ii. For what type(s) of triangles are the incenter, the centroid, and a vertex of the triangle WXY collinear?

      iii. For what type(s) of triangles are the incenter and centroid located at exactly the same point?

3. Circumcenter:

   (a) To form the circumcenter (C), select the side lengths of the triangle and create the midpoints. From each midpoint, construct the perpendicular line through that point by using the Construct menu. Where these intersect, create the intersection point and label the new point "C". Hide the perpendicular bisectors you created so that you are left with the triangle WXY and the points M, I, and C.

   (b) Verify that C is the circumcenter by creating a circle centered at C with a radius of CA. Does the circle pass through the other vertices on the original triangle? If so, hide the circle.

   (c) Move the points of the triangle WXY around to answer the following questions. Note you may need the lengths XY, XW, and WY, along with the angles across from them to see what types of triangles you have (acute, obtuse, equiangular, right):

      i. What type(s) of triangles have C in the interior?

      ii. What type(s) of triangles have C in the exterior?

      iii. Are there any triangles that have C on end edge of the triangle?

      iv. For what type(s) of triangles are I, M, C, and one other vertex collinear?

      v. For what type(s) of triangles are I, M, and C located at exactly the same point?