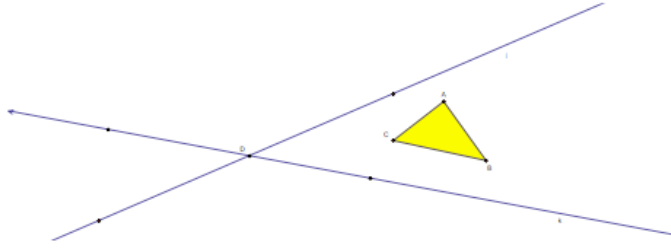# Flips and Rotations in GSP

slightly adapted from Scott Fallstrom's material

Part 1: Download and open "Reflections Lab.gsp" from the class webpage. You will find two lines intersecting and a triangle between them (see picture below).

1. Follow the directions below to reflect the triangle ABC over the line j to create $A'B'C'$ and then change the interior color of $A'B'C'$ to orange. Repeat the process and reflect the triangle $A'B'C'$ over the line k to form $A''B''C''$ and change the interior color of $A''B''C''$ to light blue.

   To reflect an object, you can use one of two ways tot tell the computer what line to reflect over:

   (a) double click on the line

   (b) click once on a line to highlight it, then from the Transform menu choose "Mark Mirror".

   Now that the line is stored in the computer's memory, you need to tell the computer what to reflect. Highlight the entire triangle (*all* the points, lines, and interior), and choose Transform-Reflect.

2. Recalling that reflection is a function we use function notation and record the reflection of triangle ABC over the line j by $F_j(ABC) = A'B'C'$. Use similar notation to record the transformation of $A'B'C'$ to $A''B''C''$.

3. Is there any way to rotate ABC into $A'B'C'$ around point D? Why or why not?

4. What is the measure of the acute angle formed between the lines (with vertex at D)?

5. Create circles that are centered at point D and have radius DC, DA, and DB. What do you notice about these circles? What points to they intersect on the various triangles?

6. Does it look like there is a way to rotate ABC into $A''B''C''$ around point D? Why or why not? If so, what is the measure of the angle of rotation?

7. Check to see if the rotation will work using the Mark Angle option. Click C, then D, then C'' in order. Then from the Transform menu, select mark angle. You'll see a few lines move across the screen showing you the angle you selected. Not click on point D and choose the Mark center option from the Transform menu. You have not told the computer exactly the angle to rotate around and exactly what point you want as the cneter. Click on the interior of ABC. Now from the Transform menu, choose "rotate". Did the shape rotate exactly where you wanted it to?

8. From the angle of rotation you found the previous problem, use the Calculate menu to create a ratio between the angle of rotation and the angle between the lines. Click on the angle from the previous problem, then divide it by the acute angle fromed between the lines. What is the result?

9. Click on point D and move it around. Does the ratio and/or angles change?

10. Click on the yellow triangle's interior and move it around. Does the ratio and/or angles change now?

11. Click on one of the points on the lines (not point D) and move it around. Does the ratio and/or angles change now? Explain what you see.

12. *Justify* your above observation. (Note, to help justify you may want to look for some relationship between the angles CDE, EDC'? Is there a relationship between the angles $C'DF$ and FDC''? Use these to make an observation about the ratio you observed between CDC'' and EDF.)

13. Now click on one of the points on the lines (not point D) and move it until the lines appear parallel. What happened to the circles you had drawn before? Why?

14. Is it possible that for *any* two lines, reflecting across both lines (one at a time) will give the same final result as a rotation?

Part 2: Make a new sketch by choosing new sketch from the File menu. The following steps are intended to help you tile or tessellate the plane with a staring shape of your choosing. Feel free to use the constructions available in GSP in other ways then those outlined below if you can tile the plane.

1. Create any three points and construct the line segments between them. Once finished create the midpoints of the sides but don't label them. Pick any one midpoint and make it the center of rotation (double-clicking on it). Select the entire shape by clicking-and-dragging over the entire triangle. Use the Transform menu to Rotate the shape around the point with an angle of 180°. What kind of quadrilateral did you construct? *Justify* your claim.

2. Double click on a different midpoint, select the entire shape (which is a quadrilateral), and rotate by 180° again. Repeat this process for the last midpoint as well. When finished, there will be 8 total congruent triangles on the screen. Click on your original triangle's vertices only by one and then select "Triangle Interior" from the Construct menu. Now, by clicking on the points in the original triangle, shrink your triangle down so all 8 triangles fit easily on the screen.

3. Click the highest point on the left of the figure, and endpoint of the line segment that extends to the left of the point you just selected. With those two points selected (in order), select the "Mark Vector" option from the Transform menu. There will be a flash of movement on the screen as the computer shows you the "vector" that you selected. This "vector" describes a direction and length which happens to be all you need for a translation.

4. Select the entire large shape and then choose "Translate" from the Transform menu. A text box will pop up. (Make sure you have the "translation vector" on top as the "marked" option.) Click the box on the bottom marked "Translate". A new shape will appear and be translated.

5. Is it possible to use similar steps as those above and use your original triangle to tessellate/tile the entire plane?

6. Move the original points in your triangle so as to consider other triangles (right, acute, isosceles, obtuse, etc). Do all of these seem to be able to tessellate/tile the entire plane?

7. On the shape you have created, click on a vertex somewhere near the middle of the shape. What is the sum of the angles at this point? How many angles are made at this point from different triangles? How do these angles relate back to the original triangle?

8. Can any triangle tile/tessellate the plane? Explain why or why not.

9. Make a New Sketch. On this new sketch, create a quadrilateral from any four points. Also construct the line segments and their midpoints. Do you think that this quadrilateral will tile/tessellate the plane?

10. Double click on any midpoint. Rotate the entire shape around that midpoint 180°. Select the midpoint on the opposite side of your original shape and rotate 180° around this point as well. Create the interior of a quadrilateral on the first quadrilateral you created. Then choose one of the remaining midpoints and rotate 180° again. Hide (Control-H) the colored region that is not the original.

11. Is it possible to use similar steps as we did with triangles to use your original quadrilateral to tessellate/tile the entire plane? Try is.

12. Move the original points in your quadrilateral around so as to consider orther quadrilaterals (rectangles, parallelograms, rhombi, etc). Do all of these seem to be able to tessellate/tile the entire plane?

13. Some quadrilateral vertex points are completely surrounded and enclosed. Choose a vertex point on your original shape that is completely surrounded. What is the sum of the angles at this point? How many angles are made at this point from different quadrilaterals? How do these relate back to the original shape?

14. Can any quadrilateral tile/tessellate the plane? Explain why or why not. Hint: consider drawing a diagonal in your quadrilateral and use a previous result you found on triangles.