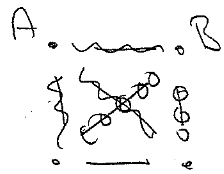
Math 213
Sections 9.1, 9.2, & 9.3

Key

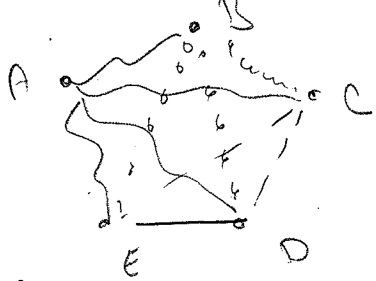
Form yourselves into groups of four to answer the following questions. Turn in one copy for each group with all the group member's names on it. This worksheet is due Tuesday 8/18 by 4:30 pm in my box.

1. Use a geometric model to find the number of handshakes that take place at a (small) party of 4 people if each person shakes hands with every one else. Hint: think about people as points and handshakes as lines.



$$\underbrace{3}_{\text{(from A)}} + \underbrace{2}_{\text{(from B)}} + \underbrace{1}_{\text{(from C)}} = 6$$

2. Repeat exercise 1 with 5 people.



$$\underbrace{4}_{\text{(from A)}} + \underbrace{3}_{\text{(from B)}} + \underbrace{2}_{\text{(from C)}} + 1 = 10$$

3. Repeat the exercise with 25 people.

$$\underbrace{24}_{\text{(from 1st person)}} + \underbrace{23}_{\text{(from 2nd person)}} + \underbrace{22}_{\text{(from 3rd person)}} + \cdots + \underbrace{2}_{\text{(from 2nd to last person)}} + 1 = \frac{24 \cdot 25}{2} = 12 \cdot 25$$

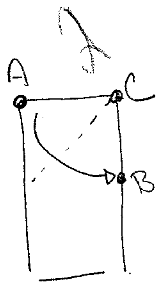
$$= 300$$

$$\begin{array}{r} 25 \\ \times\ 12 \\ \hline 50 \\ 250 \\ \hline 300 \end{array}$$

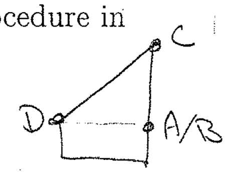
4. Generalize the exercises above to a party with n people. Give full and clear justification for your formula.

The first person shakes hands with everyone so there are $n-1$ handshakes.
The second person already shook hands with the first person so needs to only shake hands with the remaining $n-2$.
Similarly the third person already shook hands with the first and second person at the party leaving only $n-3$ people to shake hands with.
This continues to the second to last person who shakes hands with 1 person.
The total # of handshakes is thus: $(n-1)+(n-2)+\dots+2+1 = \frac{n(n-1)}{2}$ where the closed form comes to us via way of Math 211.

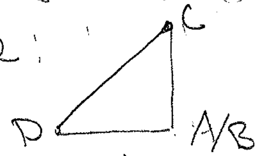
5. Question 12 from 9.2 on page 598



(a) Fold a rectangular piece of paper to create a square. Describe your procedure in writing. Explain why your approach creates a square.



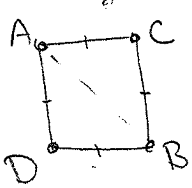
Fold on the dotted line so that $\overline{AC}$ lies on $\overline{BC}$. Next fold over DA. What remains is a triangle:



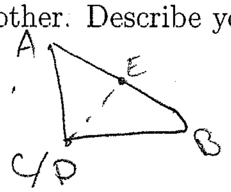
Note this implies $\angle A \cong \angle B$ & since $\angle A = 90 \Rightarrow \angle B = 90$.
Also we know $\overline{AC} \cong \overline{BC}$ & $\overline{DA} \cong \overline{DB}$
Unfold the triangle to

& fold again along $\overline{AB}$
The resulting triangle implies $90 \cong \angle C \cong \angle D$ & $\overline{AD} = \overline{AC}$. Thus all of the angles are 90° & the sides are the same length.

(b) Crease the square from the above so that the two diagonals are shown. Use paper folding to show that the diagonals of a square are congruent and perpendicular and bisect each other. Describe your procedure and explain why it works.

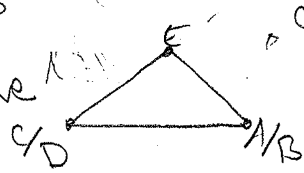


Fold along AB.

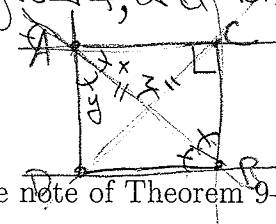


Fold A to B (along C/D E (so we are halving $\overline{AB}$)

We have



Note we can fold C/D onto A/B & see $\overline{C/E} \cong \overline{AE} \Rightarrow \overline{CD} = 2\overline{CE} \cong 2\overline{AE} = \overline{HB}$
So the diagonals are congruent, and bisect eachother.



Notice also the folding implied $x \cong y$ in the picture to the right. AIA $\Rightarrow x = y$.
Examining $\triangle \Rightarrow x = 45°$ & $z = 90°$.

& AIA wrt AC & DB

6. Try the "Now try this 9-10" on page 604, parts a, b and c. Take note of Theorem 9-5 that follows from your work. Answer provided in back of book.