

Note: This is a practice final and is intended only for study purposes. The actual exam will contain different questions and perhaps have a different layout.

1. ☐ TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

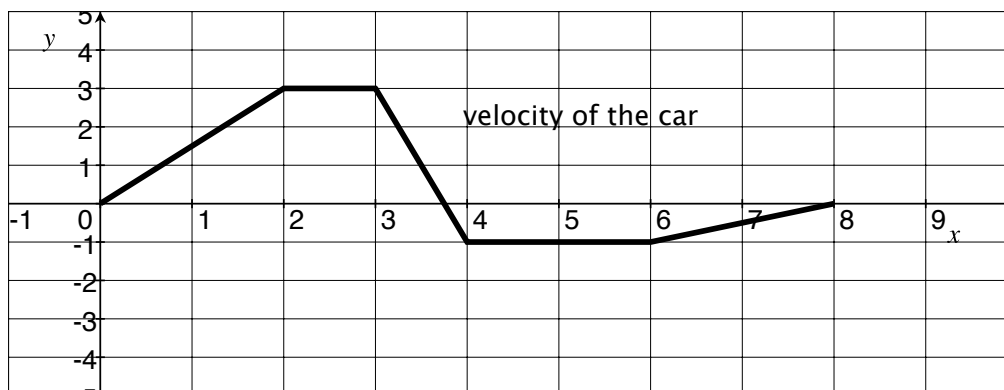
T   F    $\frac{x+h}{2x} = \frac{1+h}{x}$

T   F    $\sqrt{x^2 + h^2} = x + h$

T   F    $\frac{d}{dx}\left(\frac{1}{x}\right) = -1$

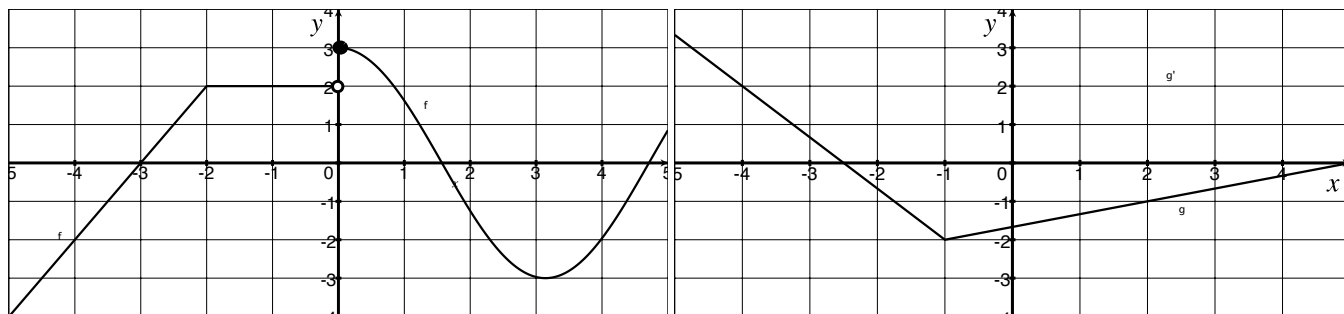
Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. The following is a graph recording the velocity of a car,  $v(x)$  (in ten's of miles per hour) as a function of 10 minute intervals,  $x$ .



- (a) Explain what  $v'(x)$  is in physical terms. Consider explaining specific examples like  $v'(1)$  or  $v'(3.5)$ .
- (b) Explain what  $\int_0^t V(x) dx$  is in physical terms. Consider explaining specific examples like when  $t$  is 3 or when  $t$  is 5.

3. Let  $f$  be the function whose graph is on the right and  $g$  be the function whose graph is on the left.



- (a) [10] Find the following (if they exist):

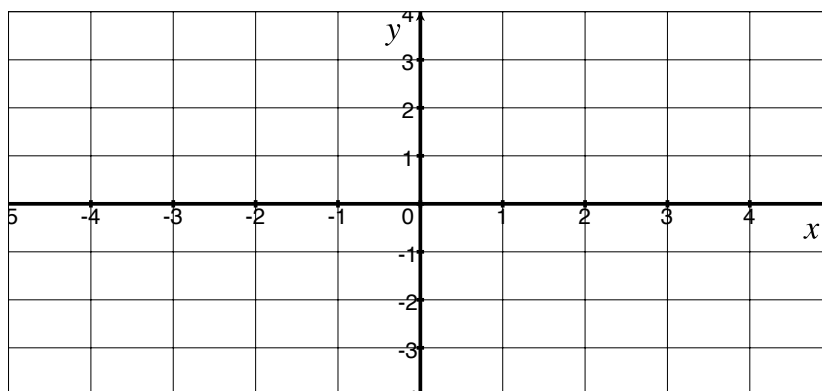
$$g(-4)$$

$$g'(-4)$$

$$(g \circ f)'(-4)$$

$$\left(\frac{f}{g}\right)'(-4)$$

- (b) [3] *Sketch* the graph of  $g'$ .



4. If  $f$  is a function defined on the interval  $[-10, 10]$ , explain in elementary terms what exactly  $f'(3)$  is.

5. Find  $\frac{dy}{dx}$  for each of the following:

(§3.1 #22)  $y = \sqrt{\frac{1}{x^3}}$

(§3.2 #5)  $y = 2^x + \frac{2}{x^3}$

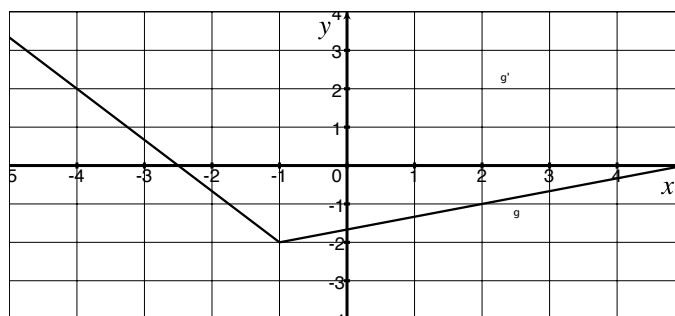
(§3.3 #17)  $y = 5e^{5x+1}$

(§3.4 #17)  $y = x \ln(2x + 1)$

6. The total cost to produce  $q$  hundred units is  $C(q) = q^2 \ln(q) - q \sin(q) + 2$ .
- (a) Find the cost of producing 150 units.
  - (b) Find the average cost of producing 150 units.
  - (c) Find the marginal cost of producing 150 units.
7. [8] A commercial cattle ranch currently allows 20 steers per acre of grazing land; on the average its steers weight 2000 lb at market. Estimates by the Agriculture Department indicate that the average market weight per steer will be reduced by 50 lbs for each additional steer added per acre of grazing land. How many steers per acre should be allowed in order for the ranch to get the largest possible total market wight for its cattle?

8. If  $f$  is a function defined on the interval  $[-10, 10]$ , explain in elementary terms what exactly  $\int_{-10}^3 f(x) dx$  is.

9. Let  $g$  be the function whose graph is given below.



- (a) Approximate  $\int_{-4}^2 g(x) dx$  using left-hand approximations and six rectangles.
- (b) Find  $\int_{-4}^2 g(x) dx$  exactly.
10. [5] The graphs of  $JIM(t)$  and  $JOHN(t)$  below trace the velocity of Jim and John respectively from time 0, measured in minutes. Explain what the physical meaning of  $\int_0^5 JIM(t) - JOHN(t) dt$  is.

11. *Carefully* write down the Fundamental Theorem of Calculus.

12. Find the following:

$$(\S 7.3 \text{ \#18}) \quad \int_0^{\frac{\pi}{4}} \sin t + \cos t \, dt$$

$$(\S 7.3 \text{ lecture}) \quad \int_1^2 \frac{x^3 + 6\sqrt{x}}{3x} \, dx$$

$$(\S 7.3 \text{ \#13}) \quad \int_1^2 \frac{1}{x} \, dx$$

$$(\S 7.3 \text{ 21b}) \quad \int_0^{\frac{\pi}{4}} \tan x \, dx$$

13. (§3.4 #36) Find the equation of the tangent line to  $f(x) = \frac{2x-5}{x+1}$  at the point at which  $x = 0$ .

14. A question involving the presentations on Monday 2/8.