Group Quiz 5
TQS 211

Key

Show *all* your work (algebraically or geometrically) for each and simplify. No credit is given without supporting work.

1. Consider $f(x) = x + 2\sin x$.

   (a) [2] Find the exact values of all the critical points of the graph of f.

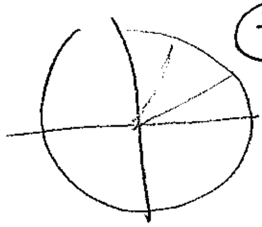
$f'(x) = 1 + 2(\sin x)'$

$= 1 + 2\cos x$ +.5

critical pts when $f'(x) = 0$ +.5

$1 + 2\cos x = 0$

$\Rightarrow \cos x = -\frac{1}{2}$



+.5 one sol

+.5 all of 'em

$\Rightarrow x = \frac{2\pi}{3} + 2\pi k$ for any integer k

or

$-\frac{2\pi}{3} + 2\pi k$ " "

   (b) [2] Find the exact values of all the points of inflections on the graph of f.

$f''(x) = 2(\cos x)'$

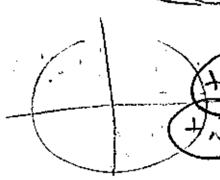
$= -2\sin x$ +.5

points of inflection may occure when

$f''(x) = 0$ +.5

or

$-2\sin x = 0$



+.5 one sol

+.5 all of 'em

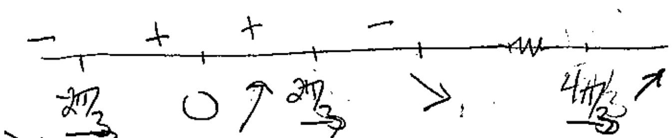
$\Rightarrow x = 0 + 2\pi k$ for any integer

or

$\pi + 2\pi k$ "

note at each point the concavity is switching.

   (c) [1] *Carefully* sketch the graph of f.

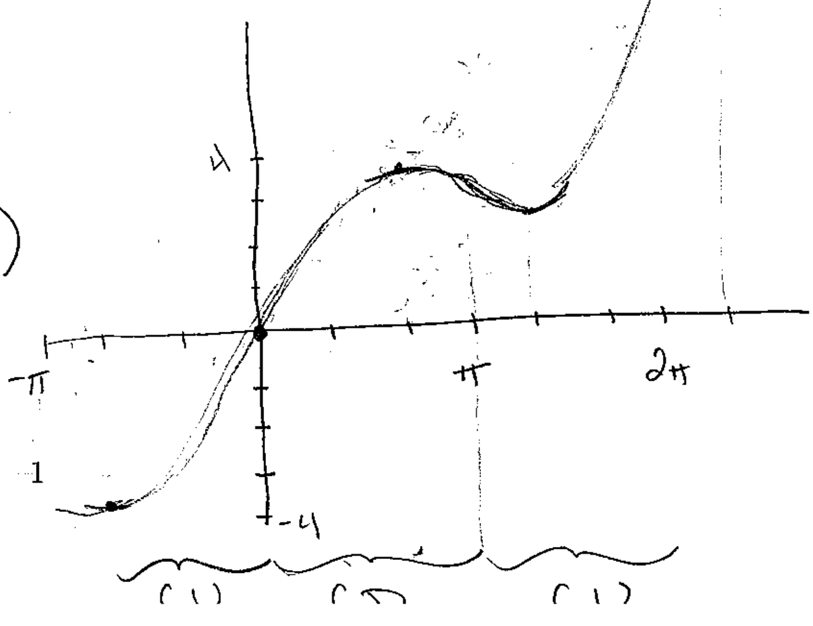


min max min

$\left(-\frac{2\pi}{3}, -\frac{2\pi}{3} - \sqrt{3}\right)$ $\left(\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3}\right)$ $\left(\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3}\right)$

$\approx$ $\approx$ $\approx$

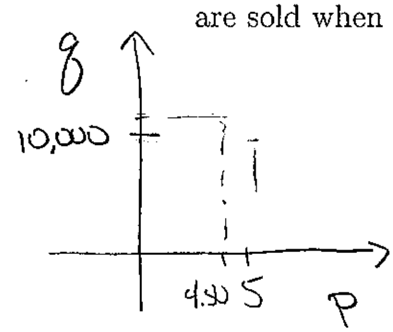
$(-2.1, -3.8)$ $(2.1, 3.8)$ $(4.2, 2.5)$



2. (a) [1] Production of an item has fixed costs of \$10,000 and variable costs of \$2 per item. Express the cost, C, of producing q items.

$$C(q) = 10000 + 2 \cdot q$$

(b) [1] The relationship between price, p, and quantity, q demand is linear. Market research shows that 10,000 items are sold when the prce is \$5 and 12,887 items are sold when the price is \$4.50. Express q as a function of price p



$(5, 10000)$

$(4.5, 12{,}887)$

$$\text{Slope} = \frac{2887}{-.5} = -5774 \quad (+.5)$$

So

$$q - 10000 = -5774(p-5)$$

$$q = -5774p + 28{,}870 + 10{,}000$$

$$= -5774p + 38{,}870 \quad (+.5)$$

(c) [3] What is the maximum profit a company with the above data can make? (Partial credit is given if you express the profit earned as a function of one variable.)

$$\text{(b)} \Rightarrow \frac{q - 38{,}870}{-5774} = p$$

$$\text{Profit} = \text{Rev} - \text{Cost}$$

$$= p \cdot q - C(q)$$

$$= \left(\frac{q - 38{,}870}{-5774}\right) q - (10{,}000 + 2q) \quad \text{in 1 variable } (+1)$$

$$= \frac{-1}{5774} q^2 + \frac{38{,}870}{5774} q - 10{,}000 - 2q \rightarrow \text{parabola opening down}$$

$$\text{Marginal Profit} = \frac{-2}{5774} q + \frac{38{,}870}{5774} - 2 \quad (+.5)$$

max when $0 = \text{Margin Profit}$ (+.5)

$$\Rightarrow 2 - \frac{38{,}870}{5774} = \frac{-2}{5774} q$$

$$\Rightarrow q = -5774 + 19435 = 13{,}661.5$$

so max Profit when $q \approx 13{,}661$ (+.5) and Profit is $\approx \$22{,}786$ (+.5)