FINAL TQS 211 Practice

Note: This is a practice final and is intended only for study purposes. The actual exam will contain different questions and perhaps have a different layout.

1. [] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

   T F $\frac{x+h}{2x} = \frac{1+h}{x}$

   T F $\sqrt{x^2+h^2} = x+h$

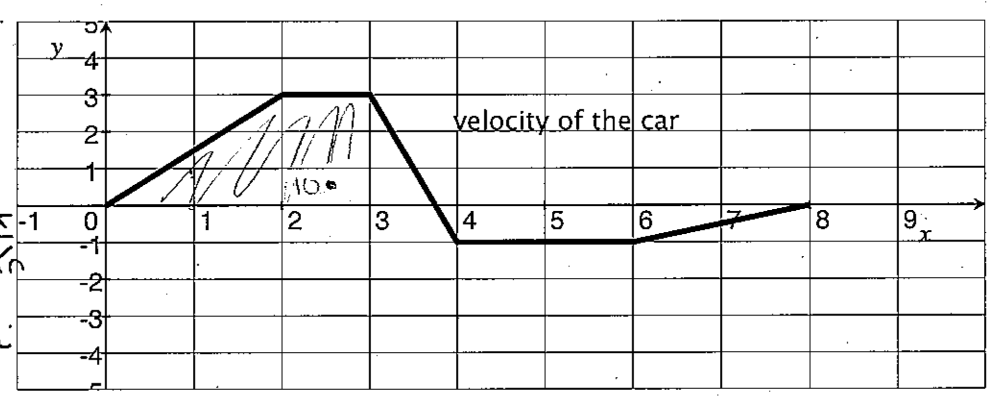
   T F $\frac{d}{dx}\left(\frac{1}{x}\right) = -1$ $\qquad \frac{d}{dx}(x^{-1}) = -x^{-2}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. The following is a graph recording the velocity of a car, $v(x)$ (in ten's of miles per hour) as a function of 10 minute intervals, $x$.

$10\frac{mi}{\cancel{hr}} \cdot 10\cancel{min} \cdot \frac{1\cancel{hr}}{60\cancel{min}}$

$= \frac{100}{60}$ mi $= \frac{5}{3}$ mi




   (a) Explain what $v'(x)$ is in physical terms. Consider explaining specific examples like $v'(1)$ or $v'(3.5)$.

   v'(x) the the rate of change of the velocity ie the acceleration. For example $v'(1) = 3/2\ m/s^2$ says the car is accelerating in the positive direction at $3/2\ m/s^2$ where as at 3.5 the car is slowing down

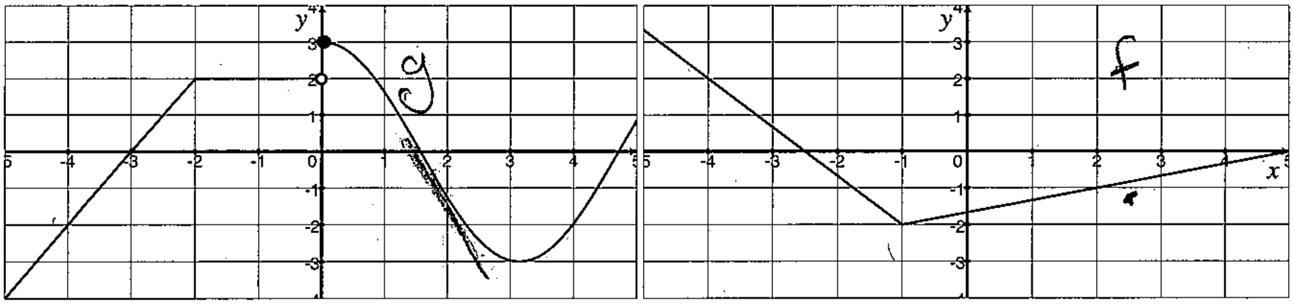
   (b) Explain what $\int_0^t V(x)\,dx$ is in physical terms. Consider explaining specific examples like when $t$ is 3 or when $t$ is 5.

   $\int_0^t v(x)dx$ is the total dist traveled from the spot at time t. For example when t is 3 the car is

   $30\frac{mi}{\cancel{hr}} \cdot 1\cancel{0}min \cdot \frac{1hr}{6\cancel{0}min} + \frac{1}{2}\, 20\,min \cdot 30\frac{mi}{hr}\frac{1hr}{60\,min}$

   $=$ 5 mi + 5 miles = 10 miles away from its starting spot.

3. Let $f$ be the function whose graph is on the right and $g$ be the function whose graph is on the left.




(a) [10] Find the following (if they exist):

$g(-4)$

$-2$

$g'(-4)$

$2$

$(g \circ f)'(-4)$

$g'(f(-4))f'(-4)$

$= g'(2) \cdot -4/3$

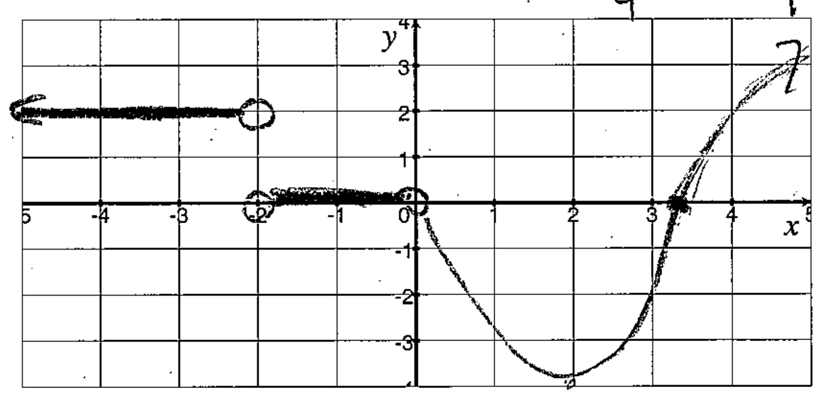
$\approx \frac{-2}{(\frac{1}{2})} \cdot -4/3 = -4 \cdot -4/3 = 16/3$

$(\frac{f}{g})'(-4) = \frac{g(-4)f'(-4) - f(-4)g'(-4)}{[g(-4)]^2}$

$= \frac{(-2)(-4/3) - (2)(2)}{(-2)^2}$

$= \frac{8/3 - 4}{4} = \frac{-4/3}{4} = \frac{-4}{12} \cdot \frac{1}{4} = -1/12$

(b) [3] *Sketch* the graph of $g'$.



note $g$ looks like a cosine function &

$d/dx(\cos x) = -\sin x$

4. If $f$ is a function defined on the interval $[-10, 10]$, explain in elementary terms what exactly $f'(3)$ is.

$f'(3)$ is the slope of the line tangent to the graph of $f$ at $x=3$.

5. Find $\frac{dy}{dx}$ for each of the following:

(§3.1 #22) $y = \sqrt{\frac{1}{x^3}}$

$y = (x^{-3})^{1/2} = x^{-3/2}$

$y' = -3/2 \, x^{-3/2 - 1}$

$= \frac{-3}{2} x^{-5/2}$

(§3.2 #5) $y = 2^x + \frac{2}{x^3}$.

$y' = 2^x \ln 2 + (2x^{-3})'$

$= 2^x \ln 2 + 2 \cdot (-3) x^{-3-1}$

$= 2^x \ln 2 - 6x^{-4}$

(§3.3 #17) y=$5e^{5x+1}$ chain rule

$f(x) = 5e^x$ $\quad f'(x) = 5e^x$

$g(x) = 5x+1$ $\quad g'(x) = 5$

$f'(g(x)) \cdot g'(x)$

$= f'(5x+1) \cdot 5$

$5e^{5x+1} \cdot 5$

$= 25e^{5x+1}$

(§3.4 #17) $y = x \ln(2x+1)$

product rule

$y' = x[\ln(2x+1)]' + (x)'[\ln(2x+1)]$

$= \underbrace{x[\ln(2x+1)]'}_{} + \ln(2x+1)$

↳ need to use the chain rul

$f(x) = \ln x$ $\quad f'(x) = 1/x$

$g(x) = 2x+1$ $\quad g'(x) = 2$

$y' = x[f'(g(x))g'(x)] + \ln(2x+1)$

$= x[f'(2x+1) \cdot 2] + \ln(2x+1)$

$= x\left(\frac{1}{2x+1}\right) \cdot 2 + \ln(2x+1)$

6. The total cost to produce $q$ hundred units is $C(q) = q^2 \ln(q) - q\sin(q) + 2$.

(a) Find the cost of producing 150 units.

$C(150) = 150^2 \ln(150) - 150\sin(150) + 2 \overset{\$}{=} 112{,}848.53$

(b) Find the average cost of producing 150 units.

ave Cost $= \frac{C(q)}{q} = \frac{q^2 \ln(q) - q\sin(q) + 2}{q}$ so $\frac{C(150)}{150} \overset{\$}{=} 752.32$

(c) Find the marginal cost of producing 150 units.

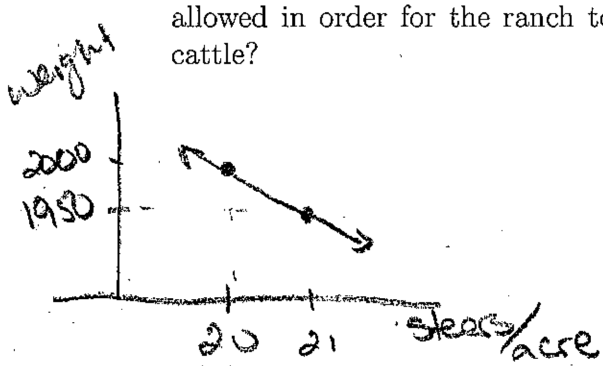
$C'(q) = [q^2 (\ln q)' + (q^2)' \ln(q)] - [q(\sin(q))' + (q)' \sin q] + \cancel{2}$

$= q^2 \frac{1}{q} + 2q \ln q - q\cos q - \sin q$

so $C'(150) = 150 + 2(150)\ln(150) - 150 \cdot \cos(150) - \sin(150)$

$\overset{\$}{=} 1549.0$ (in radians)

7. [8] A commercial cattle ranch currently allows 20 steers per acre of grazing land; on the average its steers weight 2000 lb at market. Estimates by the Agriculture Department indicate that the average market weight per steer will be reduced by 50 lbs for each additional steer added per acre of grazing land. How many steers per acre should be allowed in order for the ranch to get the largest possible total market wight for its cattle?




$x$ is the # of steers/acre

$y$ is the ave weight/steer

then we'd like to maximize

$x\left(\frac{\cancel{\text{steers}}}{\text{acre}}\right) \cdot y\left(\frac{\text{weight}}{\cancel{\text{steer}}}\right)$

$= x(-50x + 3000)$

$= -50x^2 + 3000x$

to maximize find the critical points

$0 = -100x + 3000$

$\frac{-3000}{-100} = x$

$30 = x$

Note

$y = -50x + b$

so $(20, 2000)$ is on the line

$\Rightarrow 2000 = -50 \cdot 20 + b$

$2000 = -1000 + b$

$3000 = b$

So $y = -50x + 3000$

13. (§3.4 #36) Find the equation of the tangent line to $f(x) = \frac{2x-5}{x+1}$ at the point at which $x = 0$.

14. A question involving the presentations on Monday 2/8.