EXAM 1 **TQS 211** Fall 2010

Key

~~Note: This is a practice midterm (that may be a page shorter than it ought to be) and is intended only for study purposes. The actual exam will contain different questions and perhaps a different layout.~~

1. [2] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

   T F $\sqrt{x^2+y^2} = x+y$

   T F $\frac{d}{dx}(e^1) = e$ $\quad$ $d/dx(e) = 0$

   T F Profit is equal to price minus revenue.

   Profit = Revenue − Cost.

   T F If the graph of $f'$ is increasing at $x=1$, then $f''(1) > 0$. ← Slope of line tang to $f'(x)$ at $x=1$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

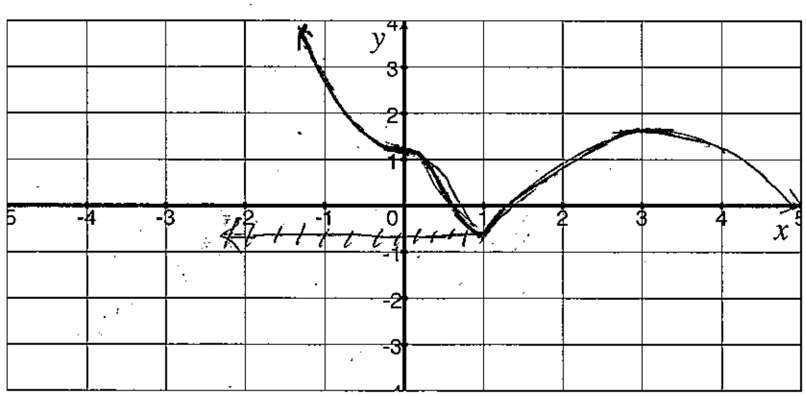
2. [5] Draw a possible graph of $f(x) = y$ given the following information:

   (a) $f'(x) > 0$ on $1 < x < 3$ $\quad$ inc.

   (b) $f'(x) < 0$ when $x < 1$ and $x > 3$ $\quad$ dec

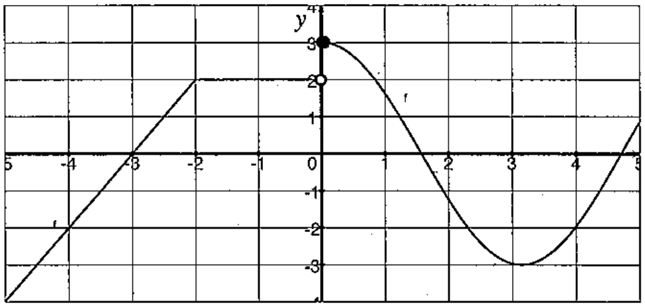
   (c) $f''(x) < 0$ on $1 < x < 3$

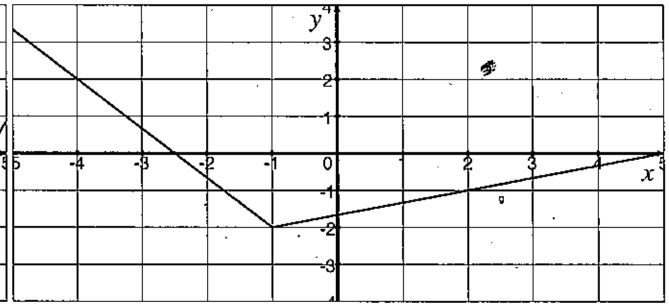
   (d) $f'(x) = 0$ at $x = 0$ and $x = 3$



and $g$ have the functions below.

3. Let $f$ be the function whose graph is below.





(a) [10] Find the following (if they exist):

$g(-4)$

$2$ (+1)

$g'(-4)$

$\frac{-4}{3}$ (+2)

about $\frac{1}{3}$ (+1)

$g(-2) = g(f(-4)) = (g \circ f)(-4)$ ~~plug in #~~ (+1)

$(g\circ f)'(-4) = g'(f(-4)) \cdot g'(-4)$ (+1) (+1)

$g'(-2) \cdot -\frac{4}{3}$ (+1)

$-\frac{4}{3} \cdot -\frac{4}{3} = \frac{16}{9}$ (+1)

f still got the last mark.

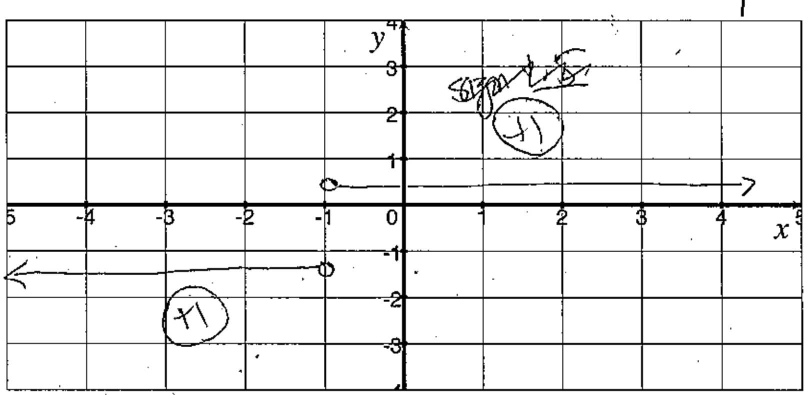
$(\frac{f}{g})'(-4)$ (+1) $\frac{g(-4) f'(-4) - f(-4) g'(-4)}{[g(-4)]^2}$ plug in # (+1)

$\frac{2 \cdot 2 - (-2)(-\frac{4}{3})}{2^2}$ (+1)

$= \frac{4 - \frac{8}{3}}{4} = \frac{12-8}{3} \cdot \frac{1}{4}$ (+1)

$= \frac{4}{3} \cdot \frac{1}{4} = \frac{1}{3}$

(b) [3] *Sketch* the graph of $g'$.



$y = \frac{1}{3}$

$y = -\frac{4}{3}$

signs 1.5 (+1)

(+1)

~~horizontal ends~~ (+.1)

Sketch (1.5)

4. The demand curve for a product is given by $q = 300 - 3p$, where $p$ is the price of the product and $q$ is the quantity consumers will buy at that price.

(a) [2] Write the revenue as a function of price.

(+1)

$Rev = p \cdot q$

$= p(300-3p) = 300p - 3p^2$

(+1)

got rid of.

(b) [3] Find the marginal revenue when the price is \$10, ~~and interpret your answer~~ in ~~terms of revenue~~.

$MR\big|_{p=10}$ $\quad dR/dp = 300 - 6p$ (+1)

$\Rightarrow dR/dp\big|_{p=10} = 300 - 6 \cdot 10$ (+1)

$= 300 - 60 = 240$

If the price is \$10 the revenue will go up about \$240 if you increase (+1) the price to 11.

(c) [4] If the marginal cost of making the product is \$20, and the business has the ability to set the price (by controlling $q$), what should the business set the price to so as to maximize profit?

Max profit when
$MC = MR$ (+1)

$20 = 300 - 6p$ (+1)

$\Rightarrow +280 = +6p$

$\$ \frac{280}{6} = p$

alg (+.5)

stated (+.5)

So about

\$46.67

$46.6$
$6 \overline{)280}$
$24$
$40$
$36$
$4$

5. [4] Let $f(x) = x^2$ and $g(x) = x^2 + 3$. What can you say about the slopes of the tangent lines to the two graphs at point $x = 1$? $x = a$, where $a$ is any value? Justify your comments.

(+1) at $x=1$, the slopes of the tang. lines to $f$ & $g$ at $x=1$ are ~~equal~~. the same (so the lines are //)

(+1) In general the slopes of the tang lines to $f$ & $g$ ~~at~~ at a fixed $x=a$ will be parallel since

$f'(x) = 2x = g'(x)$.

(+1)

stated (+1)

Simplify

6. [12] For each rule of $f$ given below, find $f'(x)$.

$f(x) = 5 \cdot 3^x + \ln x$

$f'(x) = 5(\ln 3)3^x + \frac{1}{x}$ (+1) (+1) (+1)

$f(x) = e^{5-2x}$

(+1) $g(x) = 5-2x$ $f(x) = e^x$

$g'(x) = -2$ $f'(x) = e^x$

(+1) $f'(x) = f'(g(x)) \cdot g'(x)$

$= f'(5-2x) \cdot -2$

$= -2e^{5-2x}$ (+1)

$f(x) = \frac{3x^7 - x}{\sqrt{x}} = 3x^{6.5} - x^{.5}$

$f'(x) = 3 \cdot 6.5x^{5.5} - .5x^{-.5}$

alg (+1)

pwr term by term (+2)

or

quotient rule (+1)

der of tops (+1)

got it (+1)

$f(x) = (\sqrt{3x^4 - x})e^{5-2x}$

did above

$(\sqrt{3x^4-x})'e^{5-2x} + \sqrt{3x^4-x}\,(e^{5-2x})'$ prod (+1)

$f(x) = \sqrt{x}$ $g(x) = 3x^4 - x$

$f'(x) = \frac{1}{2}x^{-1/2}$ $g'(x) = 12x - 1$

used chain inside (+1)

$\frac{1}{2}[3x^4-x]^{-1/2} \cdot (12x-1)e^{5-2x} + \sqrt{3x^4-x} \cdot -2e^{5-2x}$

der of smaller pieces (+1) (+1)

7. [5] Find the equation of the tangent line to the graph of $y = 2x^3 - 5x^2 + 3x - 5$ at $x = 1$. $\to g(x)$

looking for $y - y_1 = m(x - x_1)$ (+1)

Finding slope $m = g'(1)$ (+1)

So $g'(x) = 6x^2 - 10x + 3 \Rightarrow g'(1) = 6 - 10 + 3 = -1$ (+1)

$\Rightarrow m = -1$

Finding $x_1$ & $y_1$, if $x_1 = 1$ then $y_1 = 2(1)^3 - 5(1)^2 + 3(1) - 5$ (+1)

$= 2 - 5 + 3 - 5 = -5$

$\Rightarrow x_1 = 1$ then $y_1 = -5$

So

$y + 5 = -1(x - 1)$ (+1)