

Note! Limits will be on free!

FINAL

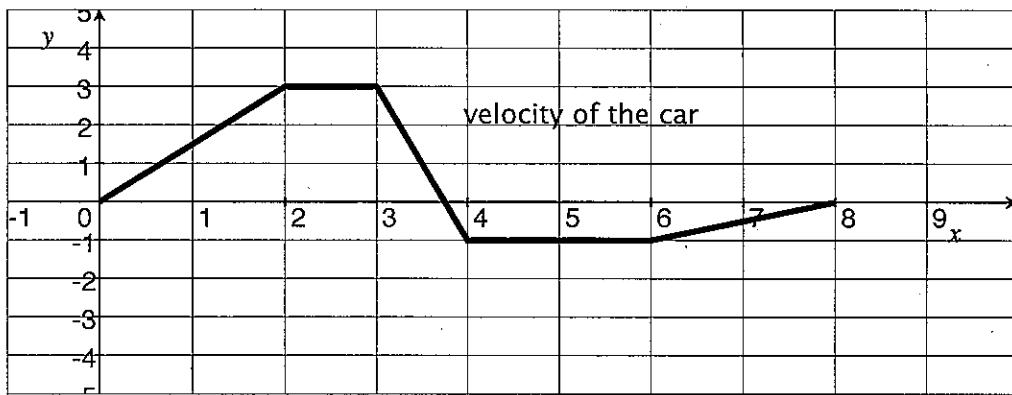
TQS 211

Practice

Note: This is a practice final and is intended only for study purposes. The actual exam will contain different questions and perhaps have a different layout.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

1. The following is a graph recording the velocity of a car, $v(x)$ (in ten's of miles per hour) as a function of 10 minute intervals, x .



- (a) Explain what $v'(x)$ is in physical terms. Consider explaining specific examples like $v'(1)$ or $v'(3.5)$.

$v'(x)$ is the rate of change of the velocity i.e. the acceleration
For example, $v'(1) = \frac{3}{2} \text{ m/s}^2$ says the car is accelerating in the positive direction at $\frac{3}{2} \text{ m/s}^2$ whereas at 3.5 , the car is slowing down.

- (b) Explain what $\int_0^t v(x) dx$ is in physical terms. Consider explaining specific examples like when t is 3 or when t is 5.

$\int_0^t v(t) dt$ is the total distance traveled from 0 to t seconds
For example, when t is 3 the car is $30 \text{ m/s} \cdot 10 \text{ min} \cdot \frac{1}{60} \text{ hr} + \frac{1}{2} \cdot 20 \text{ min} \cdot 30 \text{ m/s}$
 $= 5 \text{ mi} + 5 \text{ mi} = 10 \text{ miles away from the starting point.}$

2. (§3.4 #36) Find the equation of the tangent line to $f(x) = \frac{2x-5}{x+1}$ at the point at which $x = 0$. Looking for $y = mx+b$

$$m = f'(0)$$

$$\begin{aligned}f'(x) &= \frac{(x+1)(2x-5)' - (2x-5)(x+1)'}{(x+1)^2} \\&= \frac{(x+1)(2) - (2x-5)(1)}{(x+1)^2}\end{aligned}$$

$$f'(0) = \frac{(0+1)(2) - (2(0)-5)(1)}{(0+1)^2} = \frac{2+5}{1} = 7\text{.}$$

Note, the line passes through $(0, f(0)) = (0, \frac{2(0)-5}{0+1})$

$$= (0, -5)$$

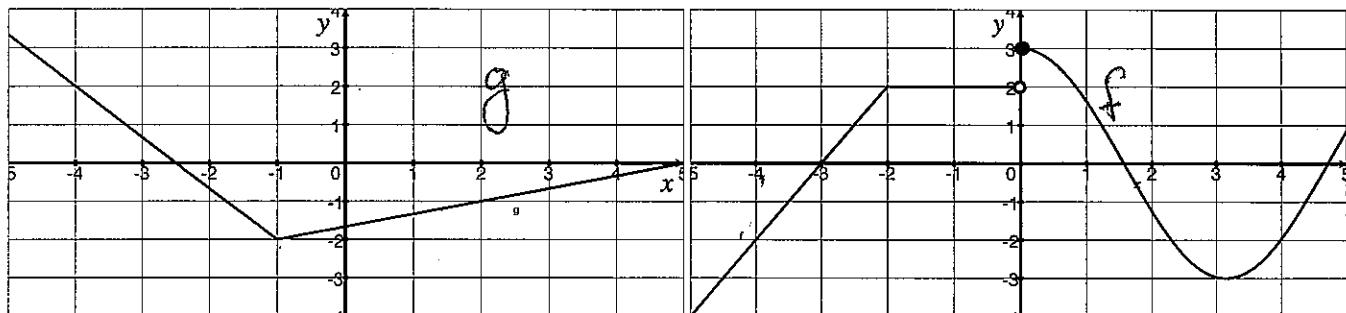
$$= (0, -5)$$

$$\text{So } -5 = f(0) + b$$

$$\Rightarrow b = -5$$

Thus $\boxed{y = 7x - 5}$

3. Let f be the function whose graph is on the right and g be the function whose graph is on the left.



(a) [10] Find the following (if they exist):

$$g(-4)$$

2

$$g'(-4)$$

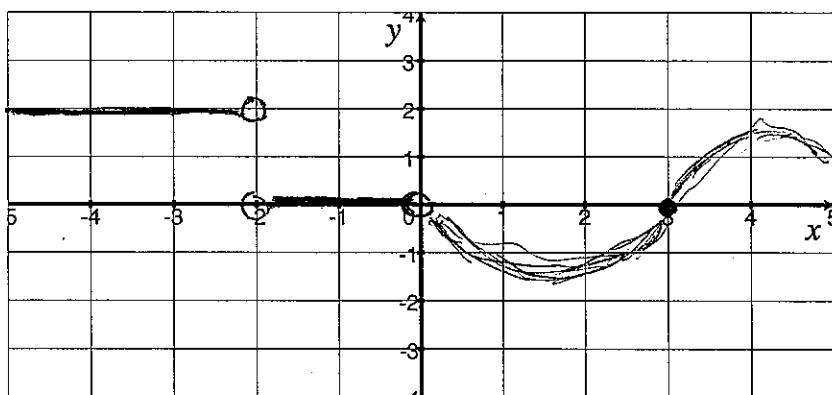
$$= -\frac{4}{3}$$

$$\begin{aligned} & (g \circ f)'(-4) \\ &= g'(f(-4))f'(-4) \\ &= g'(-2) \cdot \frac{2}{1} \\ &= -4/3 \cdot 2/1 = -8/3 \end{aligned}$$

$$\begin{aligned} & (\frac{f}{g})'(-4) = \frac{g(-4)f'(-4) - f(-4)g'(-4)}{[g(-4)]^2} \\ &= \frac{2 \cdot 2 - (-2)(-\frac{4}{3})}{2^2} \\ &= \frac{4 - \frac{8}{3}}{4} = \frac{\frac{12-8}{3}}{4} = \frac{\frac{4}{3}}{4} = \frac{1}{3} \end{aligned}$$

(b) [3] Sketch the graph of f' .

$$= \frac{1}{3}$$



4. If f is a function defined on the interval $[-10, 10]$, explain in elementary terms what exactly $f'(3)$ is.

$f'(3)$ is the slope of the line tangent to the graph of f at $x=3$.

5. Find $\frac{dy}{dx}$ for each of the following:

$$(\S 3.1 \#22) \quad y = \sqrt{\frac{1}{x^3}}$$

$$\begin{aligned} y &= (x^{-3})^{\frac{1}{2}} = x^{-\frac{3}{2}} \\ y' &= -\frac{3}{2} x^{-\frac{3}{2}-1} \\ &= -\frac{3}{2} x^{-\frac{5}{2}} \end{aligned}$$

$$(\S 3.2 \#5) \quad y = 2^x + \frac{2}{x^3}$$

$$\begin{aligned} y' &= (\ln 2) 2^x + (2x^{-3})' \\ &= 2^x \ln 2 + 2(x^{-3})' \\ &= 2^x \ln 2 + 2(-3)x^{-3-1} \\ &= 2^x \ln 2 - 6x^{-4} \end{aligned}$$

$$(\S 3.3 \#17) \quad y = 5e^{5x+1} \quad \text{chain rule}$$

$$f(x) = 5e^x$$

$$f'(x) = 5e^x$$

$$f'(g(x))g'(x)$$

$$= f'(5x+1) \cdot 5$$

$$= 5e^{5x+1} \cdot 5$$

$$= 25e^{5x+1}$$

$$g(x) = 5x+1$$

$$g'(x) = 5$$

$$y' = x \underbrace{[\ln(2x+1)]'}_{\text{need chain rule}} + (x)' \ln(2x+1)$$

need chain rule

$$f(x) = \ln x \quad g(x) = 2x+1$$

$$f'(x) = \frac{1}{x} \quad g'(x) = 2$$

$$y' = x f'(g(x))g'(x) + \ln(2x+1)$$

$$= x f'(2x+1) \cdot 2 + \ln(2x+1)$$

$$= x \frac{1}{2x+1} \cdot 2 + \ln(2x+1)$$

3

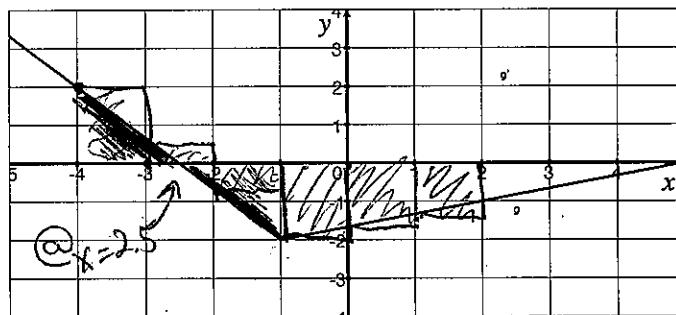
$$= \frac{2x}{2x+1} + \ln(2x+1)$$

6. If f is a function defined on the interval $[-10, 10]$, explain in elementary terms what exactly $\int_{-10}^3 f(x) dx$ is.

$\int_{-10}^3 f(x) dx$ is the signed area between $-10 \leftarrow 3$.

i.e. $\int_{-10}^3 f(x) dx$ records the total area bounded above the x -axis & below the graph of f minus the area bounded below the x -axis & the graph of f .

7. Let g be the function whose graph is given below.



$$\begin{cases} -4/3(-4) + b = 2 \\ 16/3 + b = 2 \\ b = 6/3 - 16/3 = -10/3 \end{cases}$$

$$\begin{cases} 1/3(-1) + b = -2 \\ b = -2 + 1/3 \\ b = -5/3 \end{cases}$$

(a) Approximate $\int_{-4}^2 g(x) dx$ using left-hand approximations and six rectangles.

$$2 \cdot 1 + .5 \cdot 1 -.5 \cdot 1 - 2 \cdot 1 - 1.5 \cdot 1 - 1.3 \cdot 1$$

$$= -1.5 - 1.3 = -2.8$$

(b) Find $\int_{-4}^2 g(x) dx$ exactly. note the rule of g is $\frac{1}{3}x$

$$\begin{cases} -4/3x - 10/3 & \text{if } x \leq -1 \\ 1/3x - 5/3 & \text{else} \end{cases}$$

so the x intercept happens when

$$0 = -4/3x - 10/3$$

$$10/3 = -4/3x$$

$$-10/4 = x$$

$$-2.5 = x$$

$$\begin{aligned} \text{So, } \int_{-4}^2 g(x) dx &= \underline{\frac{1}{2} \cdot 2 \cdot 1.5} - \underline{\frac{1}{2} \cdot 2 \cdot 1.5} - \boxed{\text{XXXX}} \\ &= 3/4(1.5) - 1/2 \cdot 1 \cdot 3 \\ &= -3 - 3/2 = -\frac{6}{2} - \frac{3}{2} = -\frac{9}{2} \end{aligned}$$

8. Carefully write down the Fundamental Theorem of Calculus.

If f is continuous between $[a, b]$ + F is an antiderivative of f then

$$\int_a^b f(x) dx = F(b) - F(a)$$

9. Find the following:

$$(\S 7.3 \#18) \int_0^{\frac{\pi}{4}} \sin t + \cos t dt$$

try $-\cos t + \sin t$

$$\text{check } (-\cos t + \sin t)' = -(\cos t)' + (\sin t)'$$

$$= \sin t + \cos t \quad \checkmark$$

So by FTC

$$\begin{aligned} &= \left[-\cos(t) + \sin(t) \right] \\ &\quad - \left[-\cos(0) + \sin(0) \right] \\ &= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - (-1+0) \\ &= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 1 = 1 \end{aligned}$$

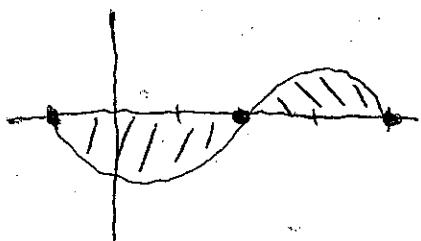
10. Let $f(x) = -(x+1)(x-2)(x-4)$

$$(a) \text{ Find } \int_{-1}^4 f(x) dx$$

$$\begin{aligned} f(x) &= -(x+1)(\overbrace{x^2 - 2x - 2x + 8}^{\text{FOIL}}) \\ &= (-x-1)(x^2 - 4x + 8) \\ &= -x^3 + 6x^2 - 8x \\ &\quad - x^3 + 6x^2 - 8x \end{aligned}$$

(b) Find the total area (not signed) bounded between the graph of f and the x -axis.

Note



$$\begin{aligned} &= \left| \int_{-1}^2 f(x) dx \right| + \left| \int_2^4 f(x) dx \right| \\ &= \left| \left[-\frac{1}{4}(x)^4 + \frac{5}{3}(x)^3 - 8(x) \right]_{-1}^2 \right| - \left| \left[-\frac{1}{4}(x)^4 + \frac{5}{3}(x)^3 - 8(x) \right]_2^4 \right| \\ &\quad + \left| \left[-\frac{1}{4}(x)^4 + \frac{5}{3}(x)^3 - 8(x) \right]_4^5 \right| - \left| \left[-\frac{1}{4}(x)^4 + \frac{5}{3}(x)^3 - 8(x) \right]_5^6 \right| \\ &\Rightarrow \left| -15.75 \right| + \left| 5.33 \right| = 21.08 \end{aligned}$$

$$(\S 7.3 \#13) \int_1^2 \frac{1}{x} dx = \int_1^2 x^{-1} dx$$

try $\ln x$

$$\text{check } (\ln x)' = \frac{1}{x} \quad \checkmark$$

So by FTC

$$\cancel{\int_1^2 x^{-1} dx}$$

$$= \ln(2) - \ln(1)$$

$$= \ln(2) - 0$$

$$= \ln(2)$$

$$\begin{aligned} \int_{-1}^4 f(x) dx &\approx \int_{-1}^4 -x^3 + 5x^2 - 2x - 8 dx \\ &\stackrel{\text{FTC}}{=} \left[-\frac{1}{4}(x)^4 + \frac{5}{3}(x)^3 - (4)^2 - 8(x) \right] \\ &\quad - \left[-\frac{1}{4}(-1)^4 + \frac{5}{3}(-1)^3 - (-1)^2 - 8(-1) \right] \\ &= -10.4167 \end{aligned}$$

11. The total cost to produce q hundred units is $C(q) = q^2 \ln(q) - q \sin(q) + 2$.

(a) Find the cost of producing 150 units.

$$C(150) = 150^2 \ln(150) - 150 \sin(150) + 2 = \$1.42 \text{ (radian mode)}$$

(b) Find the average cost of producing 150 units.

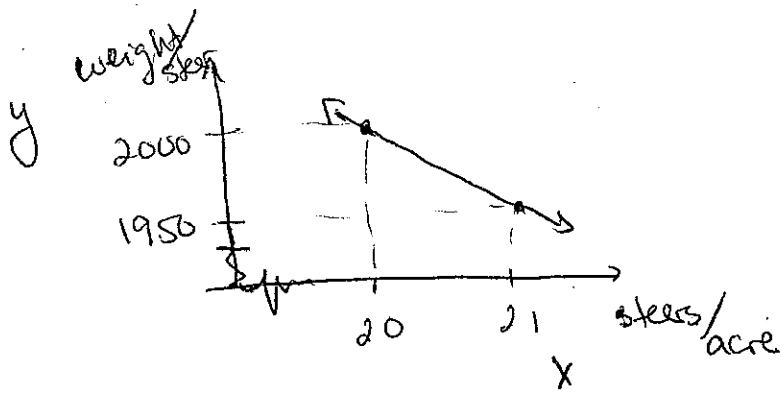
$$\text{ave cost at } 150 = \frac{C(150)}{150} = \frac{\$1.42}{150} = .00947 \text{ so } \approx .9 \text{ \$/unit}$$

(c) Find the marginal cost of producing 150 units.

$$C'(150) = ?$$

$$C'(q) = q^2 (\ln q)' + (q^2)' \ln q - (q \sin q)' + (q \sin q)' + 2' \\ = q + 2q \ln q - q \cos q - \sin q \Rightarrow C'(150) \approx 1.6 \text{ /unit}$$

12. [8] A commercial cattle ranch currently allows 20 steers per acre of grazing land; on the average its steers weight 2000 lb at market. Estimates by the Agriculture Department indicate that the average market weight per steer will be reduced by 50 lbs for each additional steer added per acre of grazing land. How many steers per acre should be allowed in order for the ranch to get the largest possible total market weight for its cattle?



line passes through $(20, 2000)$ so

$$2000 = (-50)(20) + b$$

$$2000 = -1000 + b$$

$$3000 = b$$

$$\Rightarrow y = -50x + 3000 \quad (*)$$

Maximize total weight with $30 \frac{\text{steers}}{\text{acre}}$

x is the # of steers/acre

y is the ave weight/steer

We'd like to maximize

$$x \left(\frac{\text{steers}}{\text{acre}} \right) \cdot y \left(\frac{\text{weight}}{\text{steer}} \right) = x \cdot y$$

but right now this is a function of 2 variables.

use k to substitute

$$x \cdot y = x(-50x + 3000)$$

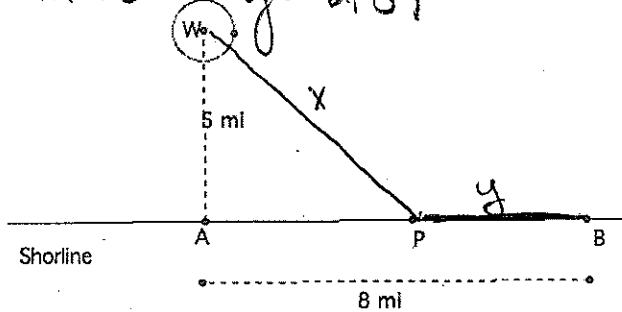
$$= -50x^2 + 3000x$$

To maximize, we find the critical points

$$0 = -100x + 3000 \Rightarrow x = 30$$

13. An offshore oil well is located in the ocean at a point W, which is 5 miles from the closest shorepoint A on a straight shoreline. The oil is to be piped to a shorepoint B that is 8 miles from A by piping it on a straight line under water from W to some shorepoint P between A and B and then on to B via a pipe along the shoreline. If the cost of laying pipe is \$100,000 per mile under water and \$75,000 per mile over land, where should the point P be located to minimize the cost of laying the pipe?

Min when $y = 2.34$



let x be the pipe laid underwater
let y be the pipe on land
measure cost in thousands of \$

$$\text{Cost} = 100x + 75y$$

We want to minimize the cost.

Right now this is a function of variables so...

So

$$\text{Cost} = 100(\sqrt{25+(8-y)^2}) + 75y$$

Find the critical points.

$$(\text{Cost})' = 100 \cdot \frac{1}{\sqrt{25+(8-y)^2}} \cdot -2(8-y)(-1) + 75$$

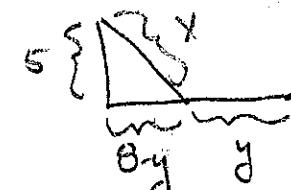
$$0 = -\frac{100(8-y)}{\sqrt{25+(8-y)^2}} + 75$$

$$-75 = -\frac{100(8-y)}{\sqrt{25+(8-y)^2}}$$

$$[-75\sqrt{25+(8-y)^2}]^2 = [-100(8-y)]^2$$

$$75^2(25+(8-y)^2) = 10000(8-y)^2$$

$$75^2 \cdot 25 + 75^2(8-y)^2 = 10000(8-y)^2$$



$$x^2 = 5^2 + (8-y)^2$$

$$\Rightarrow x = \sqrt{25 + (8-y)^2} \quad (*)$$

$$75^2 \cdot 25 = 10000(8-y)^2 - 75^2(8-y)$$

$$75^2 \cdot 25 = 4375(8-y)^2$$

$$32.1 = (8-y)^2$$

$$\sqrt{32.1} = 8-y$$

$$y = 8 - \sqrt{32.1} = 2.34$$

yes \downarrow $2.34 \nearrow 100$ Mins